

Electric Charge - It is a scalar quantity. Electric Charge is conserved, charge cannot be created nor be destroyed but can be transferred from one place to another.

S.I. unit - Coulomb

C.G.S. unit - Stat Coulomb

$$1 \text{ Stat Coulomb} = 3 \times 10^9 \text{ Coulomb}$$

Quantisation of Charge: Charge transfer from one body to another body in discrete form that is integral multiple of minimum amount of charge. $q = \pm ne$

$$(n = 1, 2, 3, \dots)$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

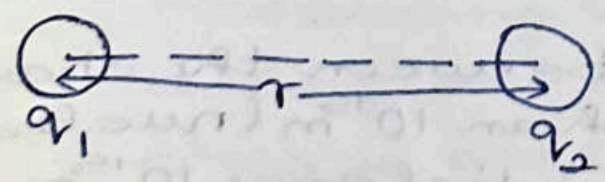
n = no. of charge

$$n = \frac{q}{e}$$

Charge show additive property i.e. charge added or subtracted algebraically.

Coulomb's Law -

Force of attraction or repulsion b/w two point charge particle in air vacuum is directly proportional to product of charges and inversely proportional to square of distance between them.



$$F \propto q_1 q_2$$

$$F \propto \frac{1}{r^2}$$

$$F \propto \frac{q_1 q_2}{r^2}$$

$$F = \frac{K q_1 q_2}{r^2}$$

$$K = \frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{\text{Nm}^2}{\text{C}^2}$$

ϵ_0 = permittivity of free space
 $= 8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$

- for air medium

$$F_a = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}$$

- for any other medium

$$F_m = \frac{1}{4\pi\epsilon} \frac{q_1 q_2}{r^2}$$

$$F_m = \frac{1}{4\pi K \epsilon_0} \frac{q_1 q_2}{r^2} \quad [\epsilon = K \epsilon_0]$$

$$\frac{F_m}{F_a} = \frac{1}{K}$$

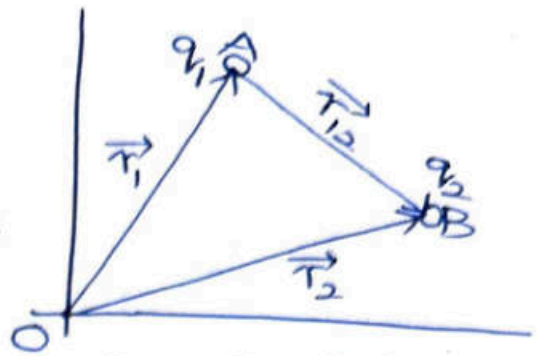
$$\boxed{K F_m = F_a}$$

Validity of Coulomb's Law -

- Coulomb's law is only obeyed for point charges, not for distribution of charges.
- Coulomb's law obey only for static charges.
- The separation between the charges must be greater than 10^{-15}m (nuclear size) because for distance 10^{-15}m strong nuclear force dominates over electrostatic force.

Coulomb's Law in Vector form-

Consider two point charges q_1 and q_2 whose position vectors are $\vec{r}_1, \vec{r}_2$.



$\vec{F}_{12}$ = Force on q_1 due to q_2

$$\begin{aligned}\vec{F}_{12} &= \frac{K q_1 q_2}{|\vec{r}_{12}|^2} \hat{r}_{12} \\ &= \frac{K q_1 q_2 (\vec{r}_1 - \vec{r}_2)}{|\vec{r}_1 - \vec{r}_2|^3}\end{aligned}$$

Apply Δ Law

$$\vec{OA} + \vec{AB} = \vec{OB}$$

$$\vec{AB} = \vec{OB} - \vec{OA}$$

$$\vec{r}_{12} = \vec{r}_2 - \vec{r}_1$$

$$\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$\vec{F}_{21}$ = Force on q_2 due to q_1

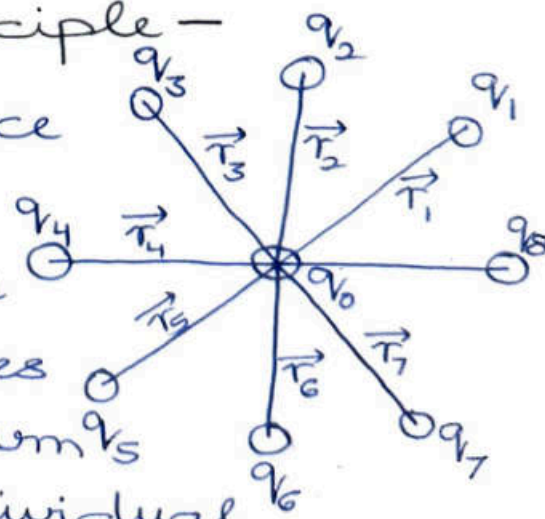
$$= \frac{K q_1 q_2}{|\vec{r}_{12}|^2} \cdot \hat{r}_{12}$$

$$= \frac{K q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$

$$\Rightarrow \vec{F}_{12} = -\vec{F}_{21} \quad \& \quad \hat{r}_{12} = -\hat{r}_{21}$$

Superposition Principle -

According to it, force acting on a test charge particle due to group of charges is equal to vector sum of force due to individual charges.



$$\vec{F}_{\text{net}} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n$$

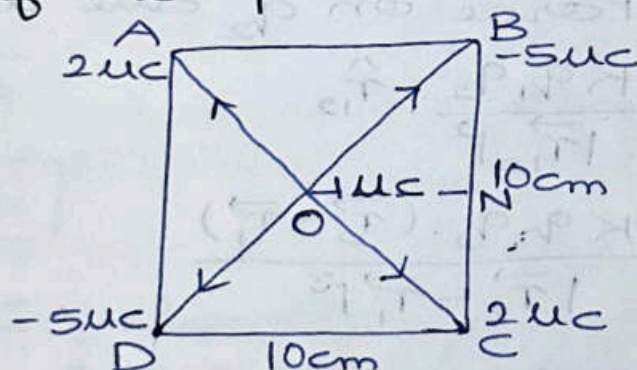
$$\vec{F}_1 = \frac{k q_0 q_1}{r_1^2} \hat{r}_1$$

$$\vec{F}_2 = \frac{k q_0 q_2}{r_2^2} \hat{r}_2$$

$$\vec{F}_n = \frac{k q_0 q_n}{r_n^2} \hat{r}_n$$

Ques Four point charges $q_A = 2\mu\text{C}$, $q_B = -5\mu\text{C}$, $q_C = 2\mu\text{C}$, $q_D = -5\mu\text{C}$ are located at the corners of a square ABCD of side 10cm. What is the force on a charge of $1\mu\text{C}$ placed at the centre of the square.

Ans



$$(ON)^2 = (OB)^2 - (BN)^2$$

$$(OB)^2 = (ON)^2 + (BN)^2 = 25 + 25$$

$$OB = 5\sqrt{2} = OC = OD = OA$$

$$\vec{F}_O = \vec{F}_{OA} + \vec{F}_{OC} + \vec{F}_{OB} + \vec{F}_{OD}$$

$$\vec{F}_{OA} = -\vec{F}_{OC}$$

$$\vec{F}_{OB} = -\vec{F}_{OD}$$

$$\Rightarrow \vec{F}_O = 0$$

Electric Field

Space around the charged body in which test charge particle experience a force of attraction or repulsion.

Electric Field Intensity

Force acting per unit test charge particle is called Electric Field Intensity.

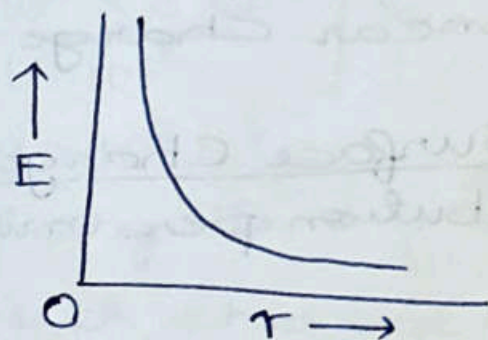
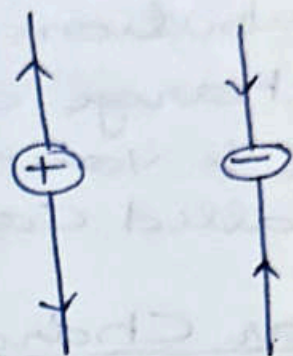
$$\vec{E} = \frac{\vec{F}}{q_0} \text{ N/C}$$

Force on test charge

$$\vec{F} = \frac{Kq q_0}{r^2} \hat{r}$$

$$\frac{\vec{F}}{q_0} = \frac{Kq}{r^2} \hat{r}$$

$$\vec{E} = \frac{Kq}{r^2} \hat{r}$$



Property of Electric Field Line -

- Electric field lines are discontinuous but b/w two point charges these are continuous.
- Electric Field Lines never intersect each other. If they intersect then there will be two directions of Electric field at a line which is not possible.
- Electric Field Lines are crowded where electric field intensity is strong and electric field lines are wider at places of weak field.

- Electric Field Line never form close loop.
- Electric Field Line outward or inward normal to a charged body.
- When two charges Q_1 and Q_2 are some distance apart then their field line may attract or repel each other depend upon nature of charges.

Distribution of Charges -

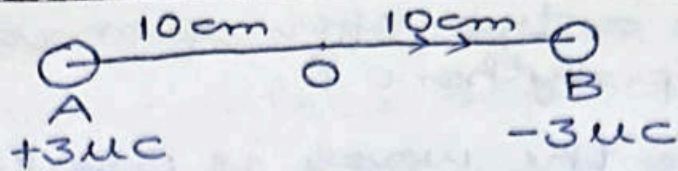
If charge distribution on length, area & volume of conductor then it is called Continuous Charge Distribution.

- Linear Charge Distribution: Charge distribution per unit length of wire is called Linear Charge Distribution. $\lambda = \frac{q}{l}$
- Surface Charge Distribution: Charge distribution per unit area of the conductor. $\sigma = \frac{q}{A}$
- Volume Charge Distribution: Charge distribution per unit volume of the conductor. $\rho = \frac{q}{V}$

Ques Two point charges $q_A = 3\mu C$ and $q_B = -3\mu C$ are located 20 cm apart in a vacuum.

- What is the electric field at the mid-point O of the line AB joining the two charges?
- If a test charge of magnitude $1.5 \times 10^{-9} C$ is placed at this point, what is the force experienced by the test charge?

Ans



Electric field at point 'O' caused by +3μC charge,

$$E_1 = \frac{1}{4\pi\epsilon_0} \cdot \frac{3 \times 10^{-6}}{(0.10)^2} \text{ N/C}$$

Electric field at point 'O' caused by -3μC charge,

$$E_2 = \frac{1}{4\pi\epsilon_0} \cdot \frac{3 \times 10^{-6}}{(0.10)^2} \text{ N/C}$$

Net Electric field at 'O'

$$E = E_1 + E_2$$

$$= \left[\frac{9 \times 10^9 \times 3 \times 10^{-6}}{(0.10)^2} \right] \times 2$$

$$= 5.4 \times 10^6 \text{ N/C}$$

along OB

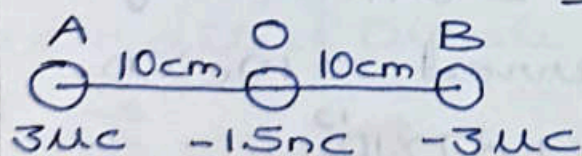
(ii) A test charge with charge $q = 1.5 \times 10^{-9} \text{ C}$ is placed at midpoint O

Let the force experienced by the test charge be F

$$\Rightarrow F = qE$$

$$= 1.5 \times 10^{-9} \times 5.4 \times 10^6$$

$$= 8.1 \times 10^{-3} \text{ N along OA}$$



Ques A polythene piece rubbed with Wool is found to have a negative charge of $3 \times 10^{-7} \text{ C}$.

(i) Estimate the number of electrons transferred

(ii) Is there a transfer of mass from wool to polythene?

Ans (i) Since the wool is positively charged and the polythene is negatively charged so we can say that few amounts of electrons are transferred from wool to polythene.

Charge on the polythene = $3 \times 10^{-7} \text{ C}$

Amount of charge on an electron
 $e = -1.6 \times 10^{-19} \text{ C}$

Let number of electron transferred from wool to polythene be n .

$$q = ne$$

$$n = \frac{q}{e} = \frac{3 \times 10^{-7}}{1.6 \times 10^{-19}} \\ = 1.87 \times 10^{12}$$

Therefore, the number of electrons transferred from wool to polythene is 1.87×10^{12}

(ii) Yes,

Mass is also transferred as an electron is transferred from wool to polythene and an electron particle have some mass.

Mass of an electron = $9.11 \times 10^{-31} \text{ Kg}$

Total mass transferred = $m_e \times n$

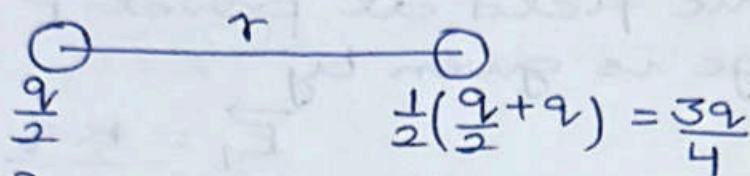
$$= 9.11 \times 10^{-31} \times 1.87 \times 10^{12}$$

$$= 1.701 \times 10^{-18} \text{ Kg}$$

Here, the mass transferred is too low that it can be neglected.

Ques Suppose the spheres A and B have their centres separated by distance 50cm and charge on each is $6.5 \times 10^{-7} \text{C}$ have identical sizes. A third sphere of same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Ans



$$F = \frac{1}{4\pi\epsilon_0} \frac{3q^2}{4 \times 2r^2} = \frac{9 \times 10^9 \times 3 \times (6.5 \times 10^{-7})^2}{(0.50)^2}$$

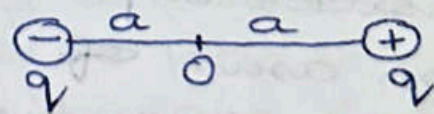
$$= 5.703 \times 10^{-3} \text{ N}$$

Electric Dipole -

If two equal and opposite charges are at some distance apart then they form electric dipole and strength of electric dipole is called Dipole Moment.

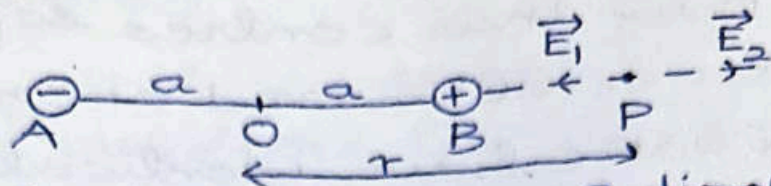
Product of any one charge and distance of separation b/w them is called Dipole Moment.

$$\vec{p} = q \cdot 2\vec{a}$$



The direction of dipole moment is from negative charge to positive charge.

Electric Field due to dipole in axial line -



Consider an electric dipole of dipole moment $\vec{p}$. A point 'P' lies along axis of dipole at a distance 'r' from the center of dipole.

$$AP = a + r$$

$$BP = r - a$$

Electric field at point P due to '-q' charge is given by

$$\vec{E}_1 = \frac{Kq}{(AP)^2} = \frac{Kq}{(r+a)^2}$$

Electric field at 'P' due to '+q' charge is given by

$$\vec{E}_2 = \frac{Kq}{(BP)^2} = \frac{Kq}{(r-a)^2}$$

$$\text{Net E.F. } \vec{E} = \vec{E}_2 - \vec{E}_1$$

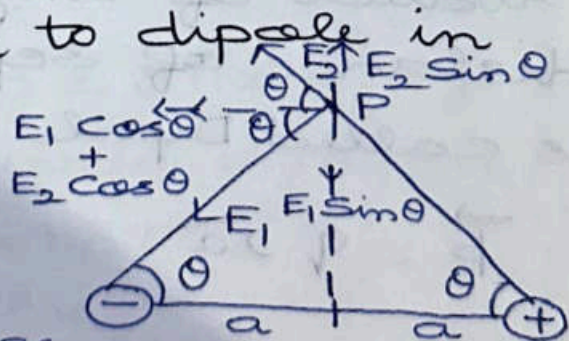
$$= Kq \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right]$$

$$= \frac{Kq \cdot 4ar}{(r^2 - a^2)^2}$$

$$E_{\text{axial}} = \frac{2Kp}{(r^2 - a^2)^2}$$

Electric field due to dipole in Equatorial Line

Consider a point 'P' on axis of dipole at a distance $\vec{r}$ from the centre of dipole.



$$\text{E.F. at P due to } -q \text{ charge } E_1 = \frac{Kq}{(\sqrt{r^2 + a^2})^2}$$

E.F. at 'P' due to +q charge

$$E_2 = \frac{Kq}{r^2 + a^2}$$

Since magnitude of E_1 and E_2 are equal. So on resolving $E_1 \sin \theta$ & $E_2 \sin \theta$ cancel each other while $E_1 \cos \theta$ & $E_2 \cos \theta$ acting along same direction.

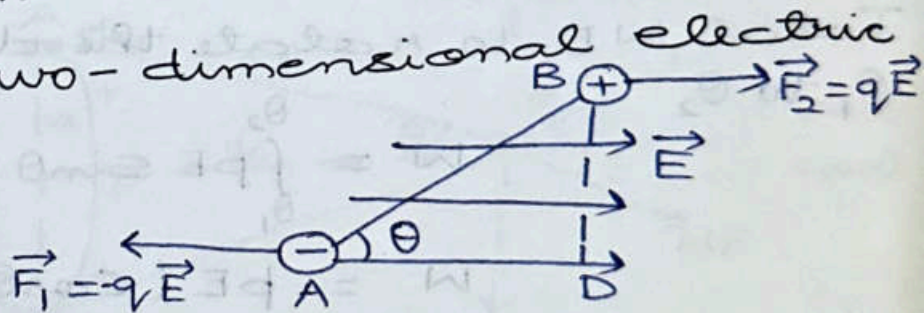
$$\text{Net E.F.} = 2E_1 \cos \theta$$
$$= 2 \cdot \frac{Kq}{r^2 + a^2} \cdot \frac{a}{\sqrt{r^2 + a^2}}$$

$$= \frac{2Kqa}{(r^2 + a^2)^{3/2}}$$

$$E_{eq} = \frac{Kp}{(r^2 + a^2)^{3/2}}$$

Note - When dipole is placed in non uniform electric field then magnitude of force change at every point and dipole never in stable eq^{mb}, it always remain unstable.

Dipole in two-dimensional electric field -



Consider an electric dipole placed in uniform electric field $\vec{E}$ make an angle ' θ ' with the field. Two equal and opposite forces acting on dipole which form a couple of force whose moment is called Torque. So, a torque acting on dipole in uniform electric field.

$$\vec{F}_{\text{net}} = 0 \text{ (in uniform electric field)}$$

Torque = moment of force

$$\tau = F_1 \times 0 + F_2 \times (2a \sin \theta)$$

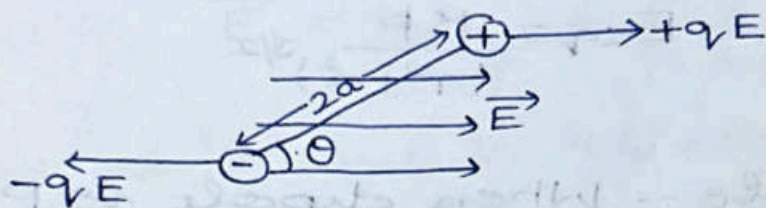
$$= qE 2a \sin \theta$$

$$= pE \sin \theta$$

$$\vec{\tau} = \vec{p} \times \vec{E}$$

Electric Potential Energy of Dipole
When a dipole is placed in uniform electric field then a torque acting on dipole which rotate the dipole in an Electric field. So some work will be done to rotate the dipole which store in form of Electric potential Energy.

$$\tau = pE \sin \theta$$



Small Work done to rotate the dipole

$$dW = \tau d\theta$$

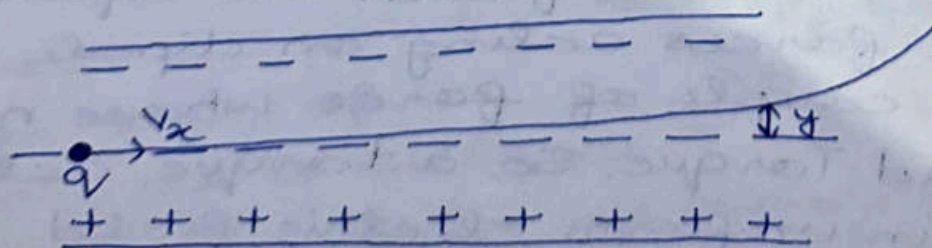
$$= pE \sin \theta d\theta$$

Total W.D. to rotate the dipole from θ_1 to θ_2

$$W = \int_{\theta_1}^{\theta_2} pE \sin \theta d\theta$$

$$W = pE (\cos \theta_1 - \cos \theta_2)$$

Behaviour of Charge particle in Uniform Electric Field -



Consider a charge particle move in electric field with initial velocity v_x

due to $\perp$ electric field charge particle deflect along y-axis.

$$F = qE$$

$$ma_y = qE$$

$$a_y = \frac{qE}{m}$$

$$x = u_x t + \frac{1}{2} a_x t^2$$

$$x = u_x t + 0$$

$$t = \frac{x}{u_x}$$

$$y = u_y t + \frac{1}{2} a_y t^2$$

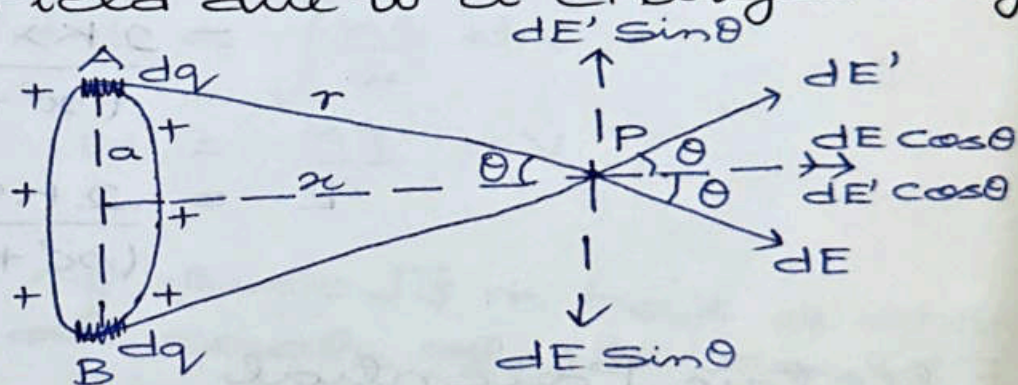
$$y = 0 + \frac{1}{2} \frac{qE}{m} \cdot \frac{x^2}{u_x^2}$$

$$y = \frac{qE}{2mu_x^2} x^2$$

$$y = K x^2$$

$$K = \frac{qE}{2mu_x^2}$$

Electric Field due to a charged body (Ring) -



Consider a uniformly charged ring of radius 'a' and linear charge density 'λ'.

$$\lambda = \frac{q}{2\pi a}$$

$$AP = BP = r = \sqrt{a^2 + x^2}$$

E.F. at 'P' due to charged element 'dq' at A is given by -

$$dE = \frac{K dq}{(AP)^2}$$
$$= \frac{K \lambda dl}{(x^2 + a^2)}$$

Similarly, E.F. at 'P' due to charged element 'dq' at B is given by

$$dE' = \frac{K dq}{(BP)^2} = \frac{K \lambda dl}{(x^2 + a^2)}$$

On resolving, $dE \sin \theta$ cancel each other and $dE \cos \theta$ acting along same dir along same axis.

$$\text{Net E.F. at 'P'} \Rightarrow E = 2 \int_0^{2\pi a} dE \cos \theta$$
$$= 2 \int_0^{2\pi a} \frac{K \lambda dl}{(x^2 + a^2)} \cdot \frac{x}{\sqrt{x^2 + a^2}}$$
$$= \frac{2 K \lambda x (2\pi a)}{(x^2 + a^2)^{3/2}}$$

$$E = \frac{2 K q x}{(x^2 + a^2)^{3/2}}$$

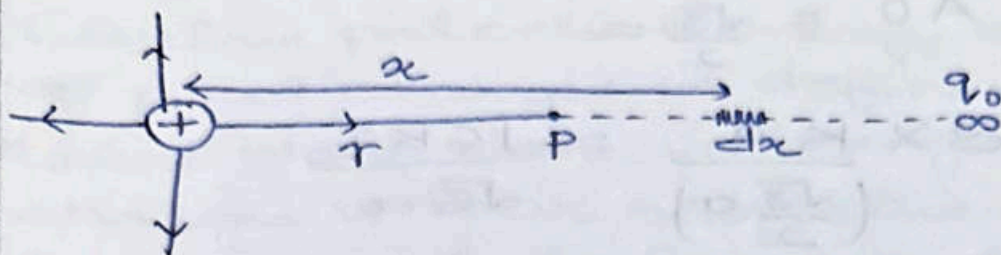
Electric Potential

Amount of W.D. to bring a unit test charge particle from ∞ to any point in the field.

$$V = \frac{W}{q_0} \quad \text{J/C}$$

Potential is a scalar quantity which may be +ve, may be -ve but have no direction.

Electric Potential due to Point Charge



Consider a charge particle and P be the point at a distance 'r' from the field of q . To calculate potential at 'P' to bring unit test charge particle from ∞ to P.

Small W.D. to bring the unit test charge particle along the field

$$dW = \vec{F} \cdot d\vec{x}$$

$$= q_0 \vec{E} \cdot d\vec{x}$$

$$= q_0 E dx \cos 180^\circ \quad (q_0 = 1)$$

$$= -\frac{Kq}{x^2} dx$$

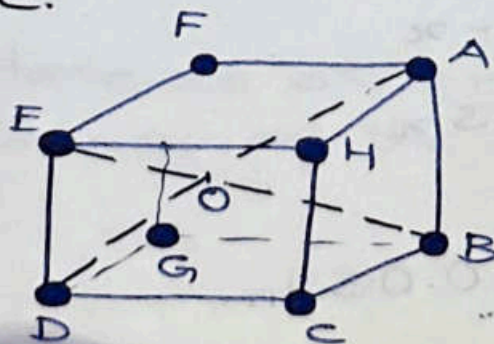
$$\int_0^W dW = -\int_{\infty}^{\frac{x}{r}} \frac{Kq}{x^2} dx$$

$$W = \frac{Kq}{r} = V$$

Ques

A cube of side $\sqrt{3}$ m has a charge $1 \mu\text{C}$ at each of its eight vertices. Find the potential and electric field at the centre of the cube.

Ans



Side of cube $a = \sqrt{3} \text{ m}$

$$AD = \sqrt{(\sqrt{2}a)^2 + a^2} = \sqrt{3} a$$

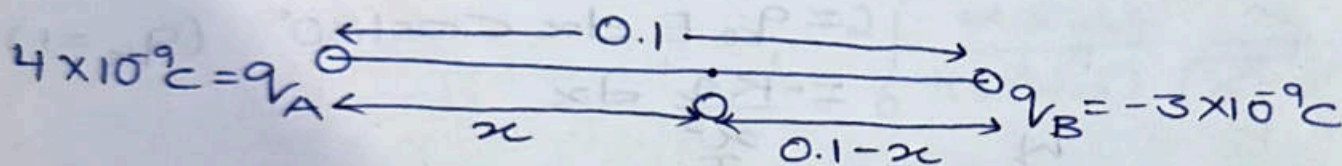
$$AO = \frac{\sqrt{3}}{2} a$$

$$V_0 = \frac{8 \times \frac{Kq}{\left(\frac{\sqrt{3}}{2} a\right)}}{\sqrt{3} a}$$

$$= 48 \times 10^3 \text{ V}$$

Ques Two charges $4 \times 10^{-9} \text{ C}$ and $-3 \times 10^{-9} \text{ C}$ are located 0.1 m apart. At what point on line joining the two charges is the electric potential zero? Take potential at infinity to be zero.

Ans



$$V_A + V_B = 0$$

$$\frac{Kq_A}{x} + \frac{Kq_B}{(0.1-x)} = 0$$

$$\frac{4 \times 10^{-9}}{x} - \frac{3 \times 10^{-9}}{(0.1-x)} = 0$$

$$\frac{4}{x} = \frac{3}{0.1-x}$$

$$0.4 - 4x = 3x$$

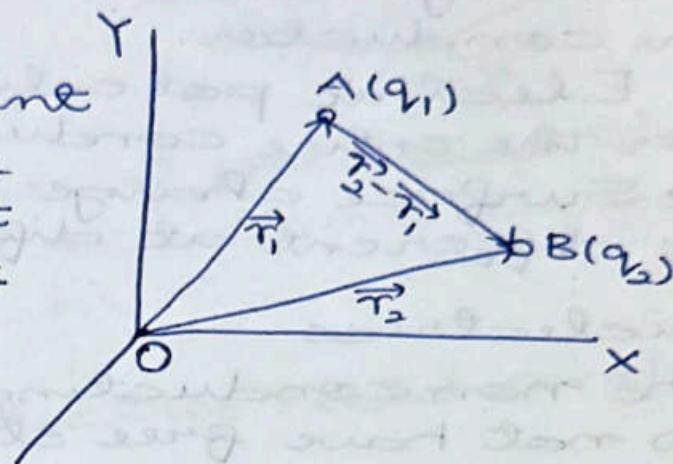
$$0.4 = 7x$$

$$x = \frac{0.4}{7} = 0.057 \text{ m}$$

Electric Potential Energy of a system of two point charges

Electric potential energy of a system of point charges is defined as the total work done in bringing these charges to their respective locations from infinity to form a system of charges.

Consider two point charges q_1 and q_2 initially lying at infinity, to a point where



When charge q_1 is brought from infinity to a position A, no work done.

When charge q_2 is brought from ∞ to the position B then the work done is $W = q_2 \times V_1$

here, $V_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{|\vec{r}_2 - \vec{r}_1|}$ (potential at B due to q_1)

$$W = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{|\vec{r}|} \quad (|\vec{r}| = |\vec{r}_2 - \vec{r}_1|)$$

This work done is stored as the potential energy of the system of two charges.

$$U = K \frac{q_1 q_2}{r}$$

Conductor in Electrostatic Field

- Net electric field intensity in the interior of conductor is zero.
- Electric field just outside the charged conductor is perpendicular to the surface of the conductor.
- Net charge in the interior of the conductor is zero.
- Charge resides on the surface of the conductor.
- Electric potential is constant for the entire conductor.
- Surface charge distribution may be different at different points.

Dielectrics

The non conducting materials which do not have free electrons are called Dielectrics.

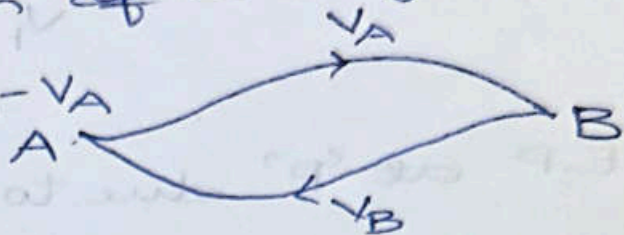
Polarisation of Dielectric

Polarisation of Dielectric is the process of inducing equal and opposite charges on the two opposite faces of the dielectric on the application of electric field.

Conservation Property of Electric Field -

W.D. to carry a unit test charge along two point in electric field does not depend upon nature of charge. It only depends upon initial and final position of charge.

$$\oint \vec{E} \cdot d\vec{l} = \frac{W_{AB}}{q_0} = V_B - V_A$$



Note -

If n liquid drops of potential ' V ' are merged then the resultant liquid drop become $V' = nV$

Relation b/w Electric Field and Potential -

$$E = -\frac{dV}{dx}$$

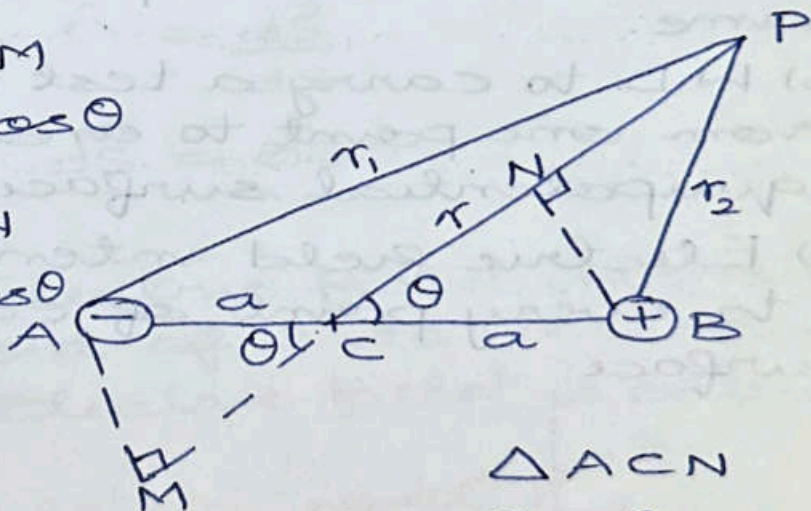
Electric Potential due to dipole at any point -

$$r_1 = AP = PM$$

$$r_1 = r + a \cos \theta$$

$$r_2 = BP = PN$$

$$r_2 = r - a \cos \theta$$



$$\triangle NCB$$

$$\cos \theta = \frac{CN}{a}$$

$$CN = a \cos \theta$$

$$\triangle ACN$$

$$\cos \theta = \frac{MC}{a}$$

$$MC = a \cos \theta$$

Consider an electric dipole of dipole moment $\vec{p}$. A point 'P' at a distance 'r' from the centre of dipole making an angle θ with the dipole.

E.P. at 'P' due to +Q charge of dipole

$$V_1 = \frac{KQ}{r_2} = \frac{KQ}{r - a \cos \theta}$$

E.P. at 'P' due to -Q charge of dipole

$$V_2 = \frac{KQ}{r_1} = \frac{KQ}{r + a \cos \theta}$$

Potential at 'P' due to dipole

$$V = V_1 + V_2$$

$$V = KQ \left[\frac{1}{r - a \cos \theta} + \frac{1}{r + a \cos \theta} \right]$$

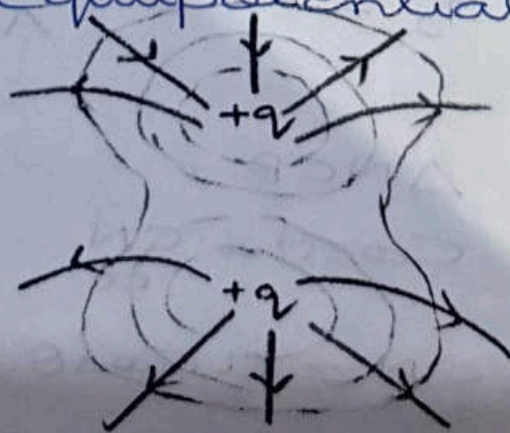
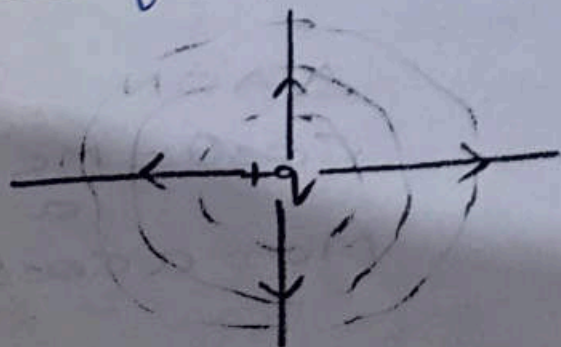
$$V = \frac{Kp \cos \theta}{r^2 - a^2 \cos^2 \theta}$$

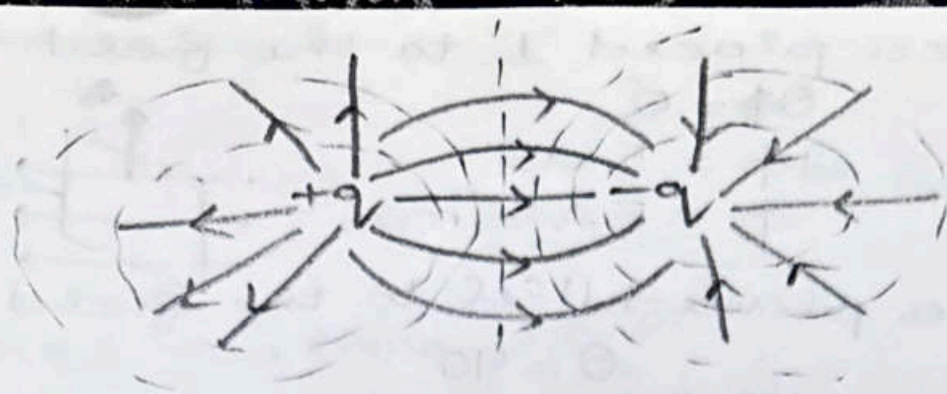
Equipotential Surface -

(a) are those surface at every point of which electric potential remain same.

(b) W.D. to carry a test charge particle from one point to another point of equipotential surface is zero.

(c) Electric field intensity is always $\perp$ to every point of equipotential surface.





Electric Flux - No. of Electric field line passing through a particular area is called Electric Flux.

It is a scalar quantity. It may be +ve, -ve or zero.

Area Vector - Area is a scalar quantity but very small area is treated as Area Vector.

Area vector is represented by outw- and normal.



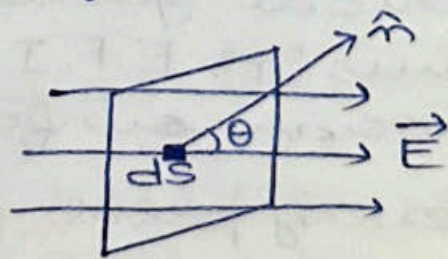
$$\hat{n} = \frac{d\vec{S}}{dS}$$

$$d\vec{S} = dS \hat{n}$$

Product of Electric field intensity and component of area vector in direction of Electric field is called Electric Flux.

$$d\phi = E \cdot dS \cos \theta$$

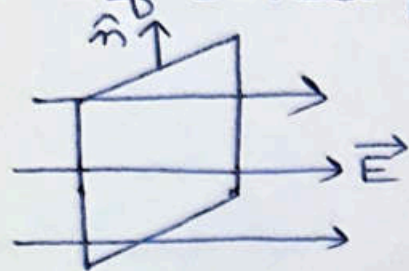
$$d\phi = \vec{E} \cdot d\vec{S}$$



(i) If area placed $\perp$ to the field
 $\theta = 0^\circ$
 $d\phi = E dS$
 $\phi = EA = \text{max.}$



(ii) If area placed $\parallel$ to the field



$\theta = 90^\circ$
 $\phi = 0$
 $\phi = \text{min}$

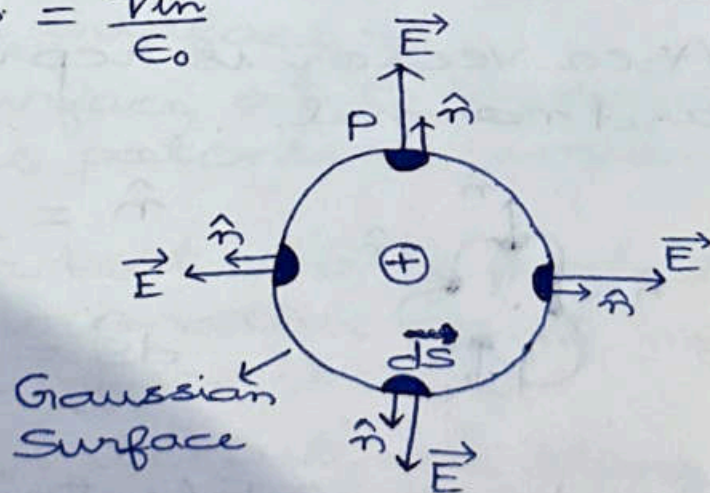
Note - If flux is -ve, it is considered as inward flux and if flux is +ve it is considered as outward flux.

Gauss Law -

Surface Integral of E.F.I. over any close path in vacuum is $\frac{1}{\epsilon_0}$ times total charge enclosed by that surface.

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{\text{in}}}{\epsilon_0}$$

PROOF



Consider a charge q and draw a spherical gaussian surface of radius ' r '. E.F.I. at every point of gaussian surface remain same.

At every point of gaussian surface E.F.I. and area vector are along

Same dir?

E.F.I. at 'P'

$$\vec{E} = \frac{Kq}{r^2} \hat{r}$$

Area vector at P

$$\hat{n} = \frac{d\vec{S}}{dS}$$

$$d\vec{S} = dS \hat{n}$$

$$\begin{aligned}\hat{r} \cdot \hat{n} &= |\hat{r}| |\hat{n}| \cos 0^\circ \\ &= 1 \times 1 \times 1 = 1\end{aligned}$$

$$\vec{E} \cdot d\vec{S} = \frac{Kq}{r^2} dS$$

Integrate both side under close path

$$\begin{aligned}\oint \vec{E} \cdot d\vec{S} &= \frac{Kq}{r^2} \oint dS \\ &= \frac{q}{4\pi\epsilon_0 r^2} (4\pi r^2)\end{aligned}$$

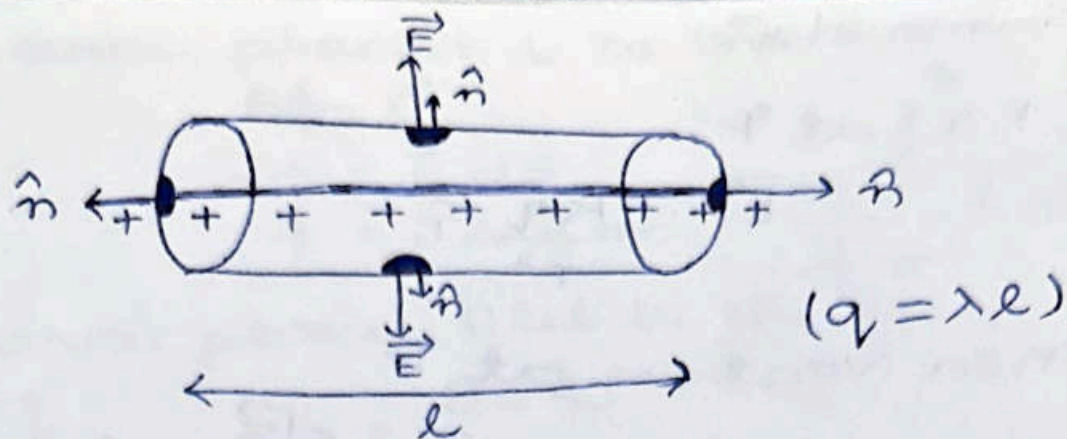
$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{in}}{\epsilon_0}$$

Note -

Gauss theorem prove only for spherical symmetric surface not for irregular surface because for irregular surface E.F.I. is not constant and area is unknown.

Application of Gauss Theorem -

(a) E.F.I. due to Long Line Charge



Consider a long line charge of length l and charge q is uniformly distributed on length of conductor.

Consider a gaussian surface around the wire which is cylindrically symmetric. At every point of gaussian surface, E.F.I. and area vector are along same dirⁿ whereas at the edge of cylinder E.F.I. and area vector are $\perp$ to each other

At edge $\Rightarrow$

$$d\phi_{\text{edge}} = E dS \cos 90^\circ + E dS \cos 90^\circ = 0$$

At Surface $\Rightarrow$

$$d\phi_{\text{surface}} = E dS \cos 0^\circ = E dS$$

According to Gauss Law

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E \oint dS = \frac{\lambda l}{\epsilon_0}$$

$$E(2\pi r l) = \frac{\lambda l}{\epsilon_0}$$

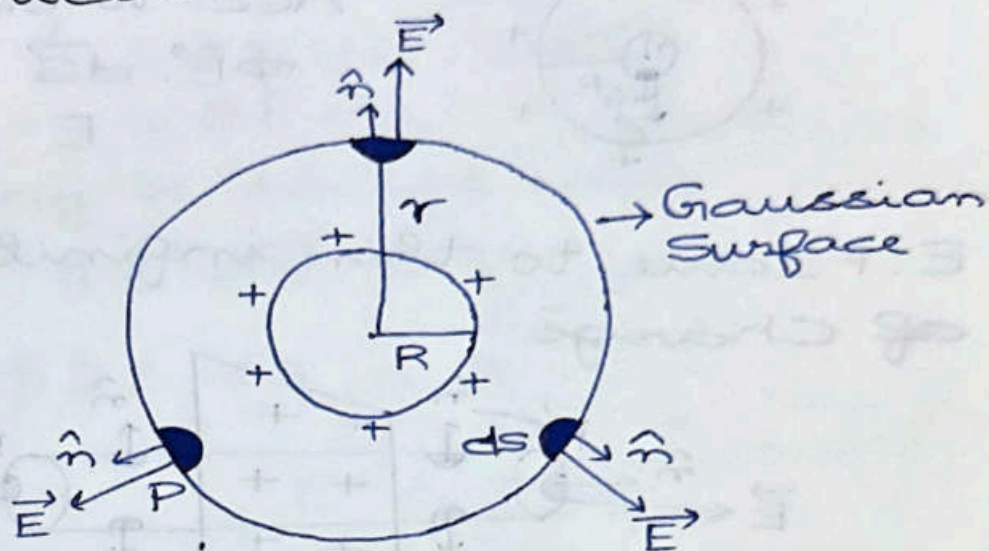
$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

$$E = \frac{2K\lambda}{r}$$

(b) E.F.I. due to Uniformly Charged spherical shell

Case-I

$$\sigma = \frac{q}{4\pi R^2}$$



Consider a point 'P' outside the spherical shell at a distance 'r'. Draw a Gaussian surface of radius 'r' which encloses the spherical charged shell. At every point of Gaussian surface, E.F.I. and area vector are along same dirⁿ.

According to Gauss theorem,

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$\oint E dS \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E \oint dS = \frac{q}{\epsilon_0}$$

$$E \cdot (4\pi r^2) = \frac{4\pi R^2 \sigma}{\epsilon_0}$$

$$E = \frac{\sigma R^2}{\epsilon_0 r^2}$$

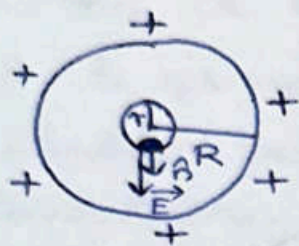
Case-II Field inside the shell

$$q_{in} = 0$$

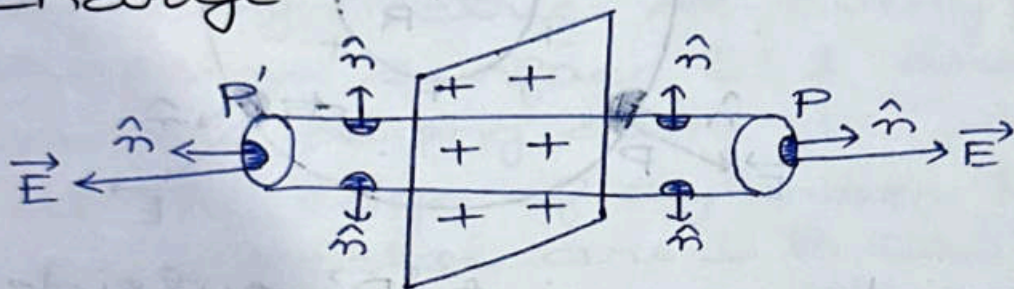
Accⁿ to Gauss Law

$$\oint \vec{E} \cdot d\vec{S} = 0$$

$$E = 0$$



E.F.I due to thin infinite sheet of Charge



Consider infinite plane sheet of charge having +ve charge distributed on area of sheet.

Assume two point at equal distance on either side of sheet and draw a gaussian surface passes through point P and P' which is cylindrically symmetric. And assume a cylinder in such a way that complete sheet is enclosed by cylinder.

At the edge of cylindrical gaussian surface, electric field and normal vector are along same dirⁿ but at curved surface, E.F. & normal

Vectors are along same dirⁿ but at curved surface, E.F. and normal vector are $\perp$ to each other.

At curved surface,

$$d\phi = E dS \cos 90^\circ = 0$$

At edge,

$$\begin{aligned} d\phi &= E dS \cos 0^\circ + E dS \cos 0^\circ \\ &= 2E dS \cos 0^\circ \\ &= 2E dS \end{aligned}$$

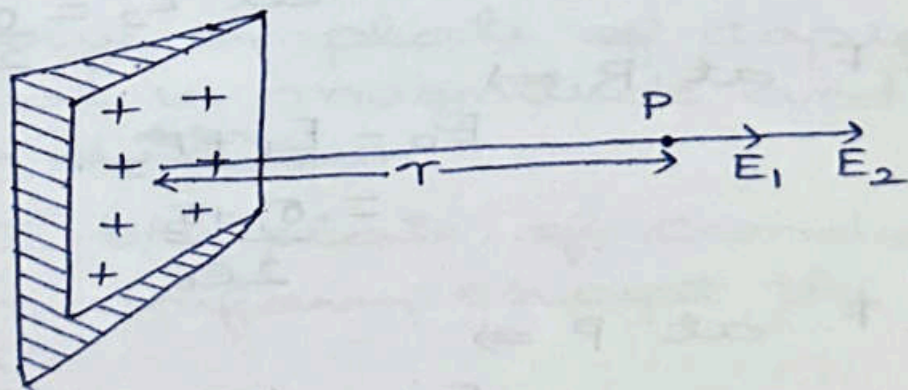
According to Gauss Law

$$\oint \vec{E} \cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$\oint 2E dS = \frac{q}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

E.F.I. due to thick plane sheet of charge -



Consider a thick plane sheet of charge of thickness 'a'. A point 'P' is at a distance 'r' from front face.

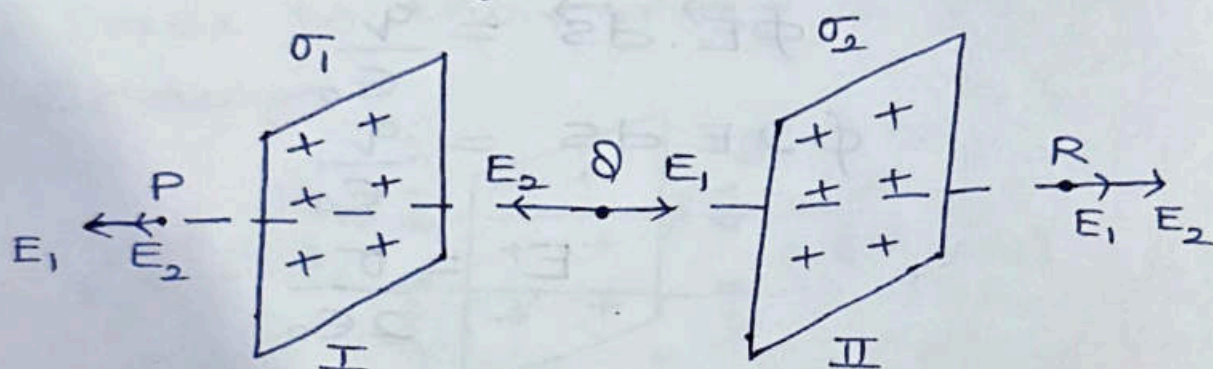
$$E_1 = \frac{\sigma}{2\epsilon_0}$$

E.F. due to sheet does not depend upon thickness so E.F. at 'P' due to back face remain acting along same dirⁿ.

$$\begin{aligned}\text{Net E.F.} &\Rightarrow E_1 + E_2 \\ &= \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}\end{aligned}$$

$$E_{\text{thick sheet}} = \frac{\sigma}{\epsilon_0}$$

Electric Field Intensity due to thin sheet of charge -



E.F.I. due to Ist sheet $E_1 = \frac{\sigma}{2\epsilon_0}$

E.F.I. due to IInd sheet $E_2 = \frac{\sigma}{2\epsilon_0}$

Net E.F. at R $\Rightarrow$

$$\begin{aligned}E_R &= E_1 + E_2 \\ &= \frac{\sigma_1 + \sigma_2}{2\epsilon_0}\end{aligned}$$

Net E.F. at P $\Rightarrow$

$$\begin{aligned}E_P &= -(E_1 + E_2) \\ &= -\frac{(\sigma_1 + \sigma_2)}{2\epsilon_0}\end{aligned}$$

Net E.F. at Q $\Rightarrow$

$$\begin{aligned}E_Q &= E_1 - E_2 \\ &= \frac{\sigma_1 - \sigma_2}{2\epsilon_0}\end{aligned}$$

Capacitor -

It is a device which can store large amount of charge and energy in small space.

Charge transfer to conductor is directly proportional to potential

$$q \propto V$$

$$q = CV$$

$$C = \frac{q}{V}$$

Principle of Capacitor -

(a) It is based upon the fact that a capacitor can store large amount of energy in small space by placing an uncharged earth conductor near a charged conductor.

(b) It is based upon the phenomenon of electrostatic induction.

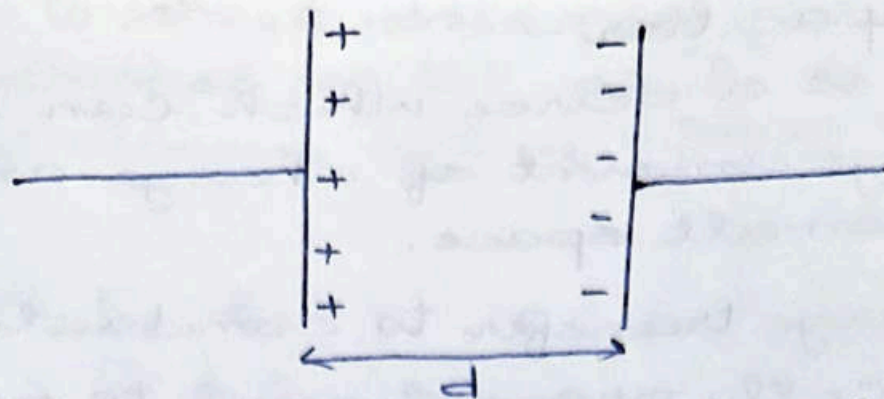
(c) Charge on plate of capacitor is equal in magnitude and opposite in ~~direction~~^{sign}.

(d) E.F.I. b/w plate of conductor remain uniform except the end of plate.

(e) If any dielectric filled in b/w the plate

$$E_m = \frac{q}{KE_0 A}$$

Parallel Plate Capacitor -



In parallel plate capacitor two parallel plane sheet of charge separated by distance 'd' and initially there is air vacuum b/w the plate. E.F.I. b/w plate of capacitor

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0}$$

If C is capacity of parallel plate capacitor.

$$q = CV$$

$$C = \frac{q}{V}$$

$$(V = E_0 d)$$

$$C = \frac{q}{E_0 \cdot d}$$

$$(E_0 = \frac{q}{A\epsilon_0})$$

$$C = \frac{\epsilon_0 A}{d}$$

Capacity of parallel plate capacitor depend upon -

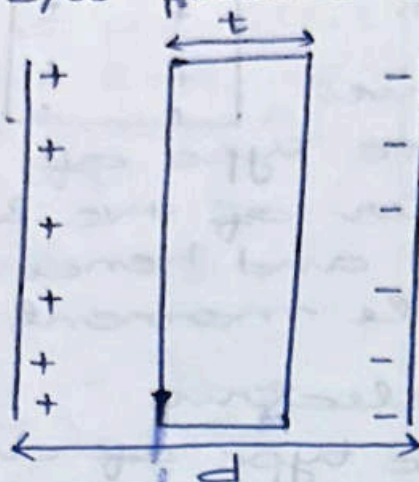
- (i) area of plate (A)
- (ii) separation b/w plate (d)
- (iii) medium b/w plate

If any dielectric medium constant K, filled b/w the plate of capacitor then capacity of capacitor become K times with respect to air vacuum.

Parallel Plate Capacitor Conduc-

-ing Slab

When a conducting slab placed partially b/w plate of capacitor.



Consider a parallel plate capacitor of each plate area A separated by 'd' distance apart. When air vacuum b/w the plate.

$$E_0 = \frac{q}{A\epsilon_0}$$

$$C_0 = \frac{\epsilon_0 A}{d}$$

When a conducting slab of thickness 't' placed b/w the plate of capacitor then inside the slab Electric field E & charge q will be zero. So, E.F. E_0 exist in $(d-t)$ distance of the plate.

Now, P.D. b/w plate of capacitor ↓

$$\begin{aligned} V &= E \cdot d \\ &= E_0 (d-t) \\ &= \frac{q}{A\epsilon_0} (d-t) \end{aligned}$$

$$C = \frac{q}{V} = \frac{q A \epsilon_0}{q (d-t)}$$

$$C = \frac{A \epsilon_0}{d-t} = \frac{A \epsilon_0}{d(1-t/d)} = \frac{C_0}{(1-t/d)}$$

Dielectric

These are special type of insulator which transmit electric effect without conduction.

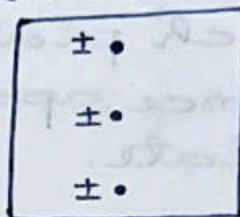


Polar Dielectric

These are those type of dielectric in which center of +ve & -ve charge are separated and hence they have non-zero dipole moment naturally.

Non Polar Dielectric

These are those type of dielectric in which center of +ve & -ve charge coincided and hence they have zero dipole moment.

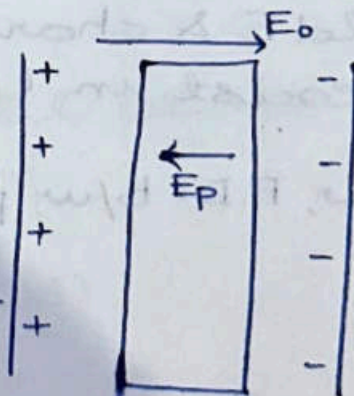


Note

When a non polar dielectric is placed in external E.F. then its charge began to separate & a stage is reached when non polar dielectric convert into polar dielectric. And it has temporary dipole moment or induce dipole moment.

Note

Due to polarisation of charge, an electric field is produced inside the dielectric which is opposite to the direction of applied field.



$$\vec{E} = \vec{E}_0 - \vec{E}_p$$

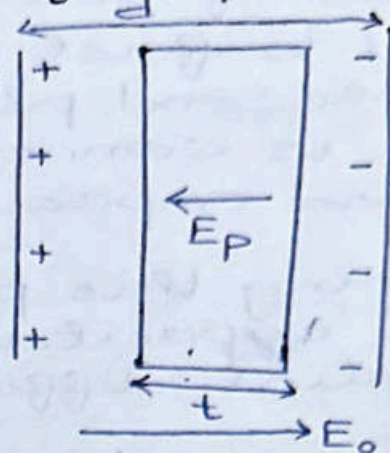
Capacitance of Parallel plate capacitor when a dielectric slab is placed b/w plate of capacitor

Consider a parallel plate capacitor of each plate area A and charge $\pm q$.

E.F. b/w plate of capacitor

$$E = \frac{\sigma}{\epsilon_0} = \frac{q}{A\epsilon_0}$$

$$C_0 = \frac{\epsilon_0 A}{d}$$



When a dielectric slab of thickness ' t ' placed b/w the plate of capacitor then due to polarization of charge, an electric field develop in dielectric slab which is opposite to the dirⁿ of applied field.

Now, E.F. ' E_0 ' exist in ' $d-t$ ' distance and ' E_p ' exist in ' t ' distance.

Potential Difference

$$\begin{aligned} V &= V_1 + V_2 \\ &= E_0 (d-t) + E_p t \\ &= E_0 (d-t) + \frac{E_0}{K} \cdot t \\ &= E_0 \left[d-t + \frac{t}{K} \right] \end{aligned}$$

$$q = CV$$

$$C = \frac{q}{V} = \frac{q A \epsilon_0}{q \left[d-t + \frac{t}{K} \right]}$$

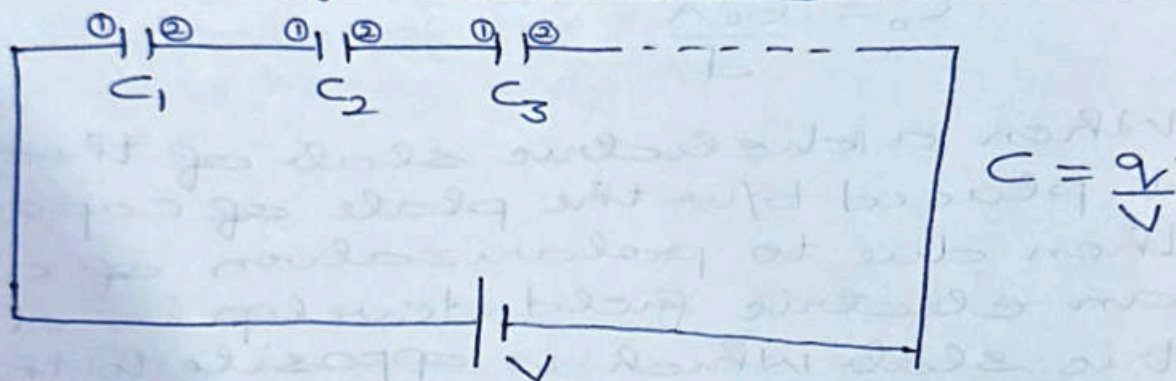
$$C = \frac{A \epsilon_0}{d \left[1 - \frac{t}{d} \left(1 - \frac{1}{K} \right) \right]} = \frac{C_0}{\left[1 - \frac{t}{d} \left(1 - \frac{1}{K} \right) \right]}$$

Series Combination of Capacitor

(a) Capacitors are said to be in series if second plate is connected to first plate of second and second plate of second capacitor is connected to first plate of third capacitor and so on.

(b) During this process, charge along each capacitor remains same, but potential difference is $V_1, V_2, \dots, V_n$.

Capacitors in Series Combination



Let no. of capacitors of capacitance $C_1, C_2, \dots, C_n$ are joined in series through a source of potential difference V .

$$V = V_1 + V_2 + V_3 + \dots + V_n$$

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \dots, \quad V_n = \frac{q}{C_n}$$

$$\Rightarrow V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3} + \dots + \frac{q}{C_n}$$

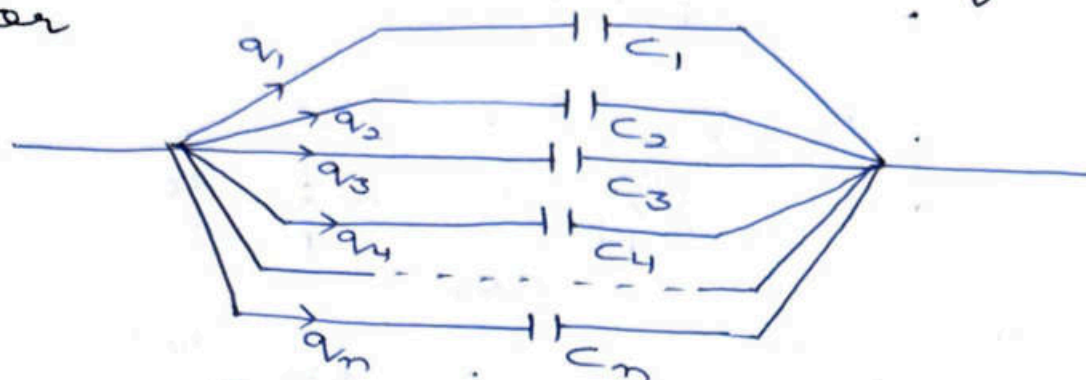
$$\frac{V}{q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}}$$

where, C_s is equivalent capacitor.

In series combination of capacitor q remain same, V different and equivalent capacitance is minimum.

Parallel Combination of Capacitor



Capacitors are said to be in parallel if one plate of each capacitor is joined at a point and second plate of each capacitor is joined at another point and these points are connected from a source. During this process, potential difference across each capacitor remains same but charge is different across different capacitors.

$$q = q_1 + q_2 + q_3 + \dots + q_n$$

$$\frac{q}{V} = C_p \Rightarrow q = C_p V$$

$$q_1 = C_1 V$$

$$q_2 = C_2 V$$

...

$$q_n = C_n V$$

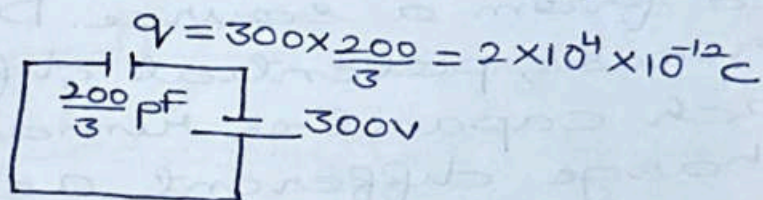
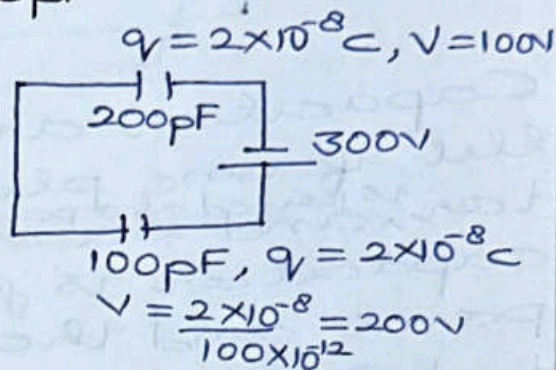
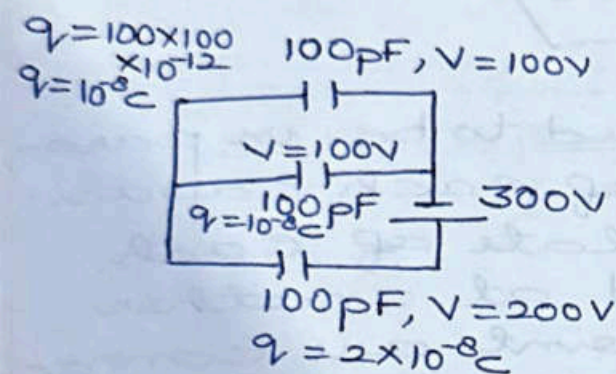
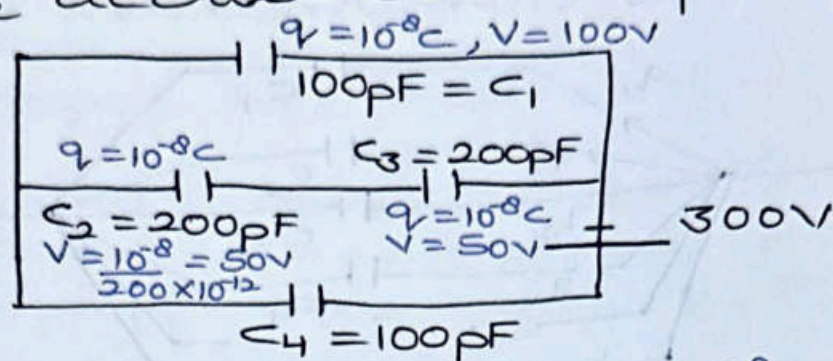
$$\Rightarrow C_p V = C_1 V + C_2 V + \dots + C_n V$$

$$\boxed{C_p = C_1 + C_2 + \dots + C_n}$$

Where, C_p is equivalent capacitance in parallel.

In parallel combination of capacitor, C is maximum, q is different and V is same.

Ques Find the charge and potential difference across each capacitor.



Ques An electrician having no. of capacitor each of $1\mu F$ and he needed an equivalent capacitor of $2\mu F$. Give an arrangement to obtain this if supply voltage is $1KV$ and one capacitor can withdraw a maximum potential of $400V$.

Ans Let n capacitor are joined in series.

$$400 \times n = 1000$$

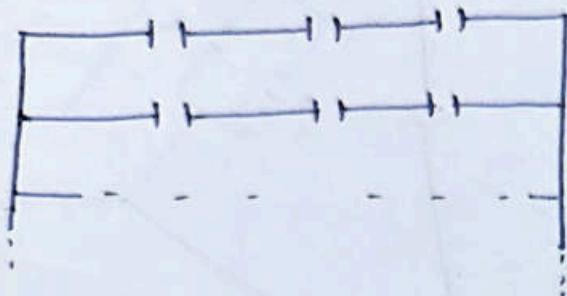
$$n = \frac{1000}{400} = 2.5 \approx 3$$

$\therefore$ 3 capacitor are in series

$$\frac{1}{C_s} = \frac{1}{1} + \frac{1}{1} + \frac{1}{1}$$

$$C_S = 1/3$$

Suppose m such rows in parallel

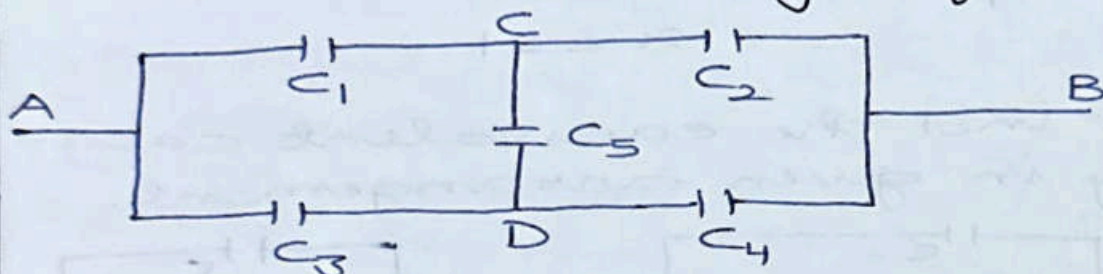


$$\frac{1}{3} \times m = 2$$

$$m = 6$$

$\Rightarrow$ No. of capacitor = 6×3 ($m \times n$)
= 18 capacitor

Wheat Stone Bridge of Capacitor



Condition of Balanced Wheat Stone Bridge -

(i) A 5 capacitor form Wheat stone bridge as shown in fig. and bridge is in balance condition when

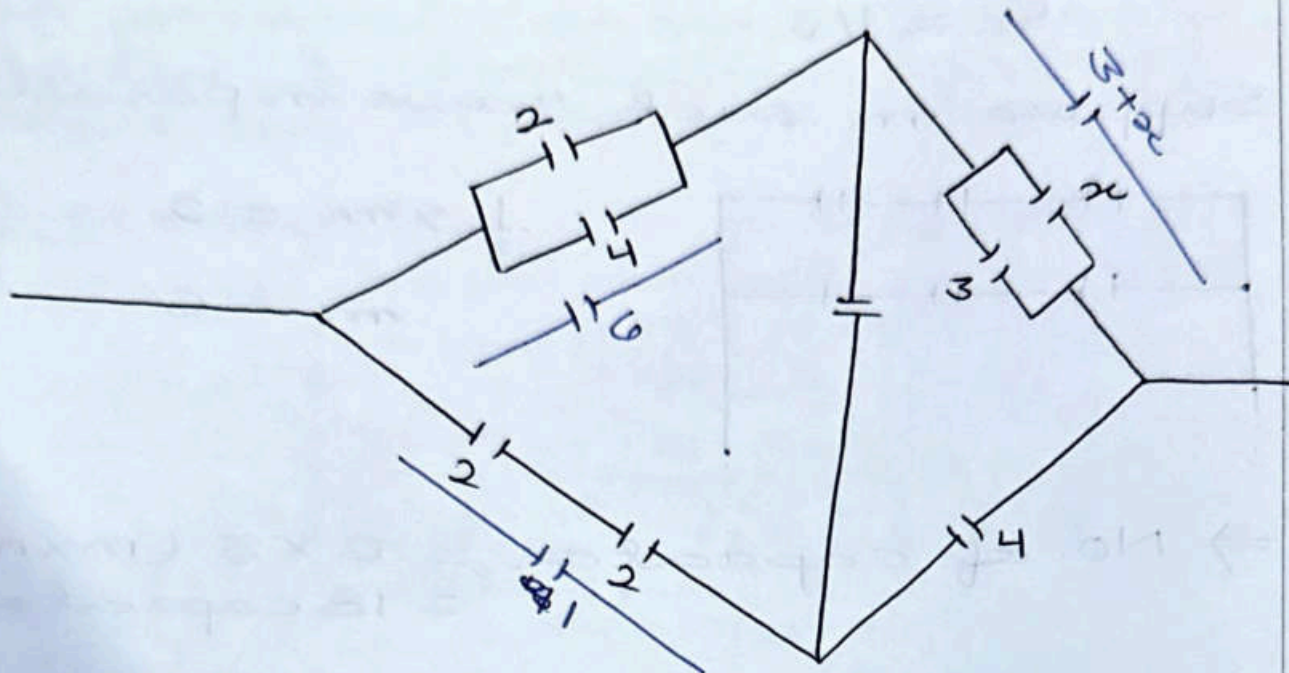
$$\frac{C_1}{C_2} = \frac{C_3}{C_4}$$

And there is no charge in C_5 and leave it.

(ii) In Balance condition, potential of point B and D is same.

$$V_B - V_D = 0$$

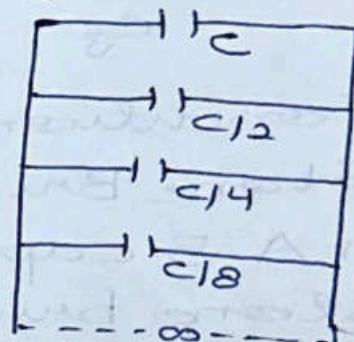
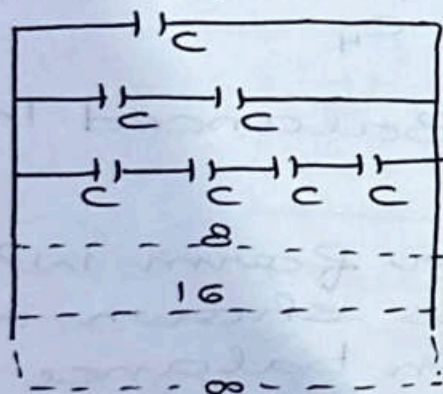
Ques This is a balanced bridge. Find the value of x .



$$\frac{6}{3+x} = \frac{6}{4} \Rightarrow 24 = 3+x$$

$$x = 21$$

Ques Find the equivalent capacitance in given arrangement.

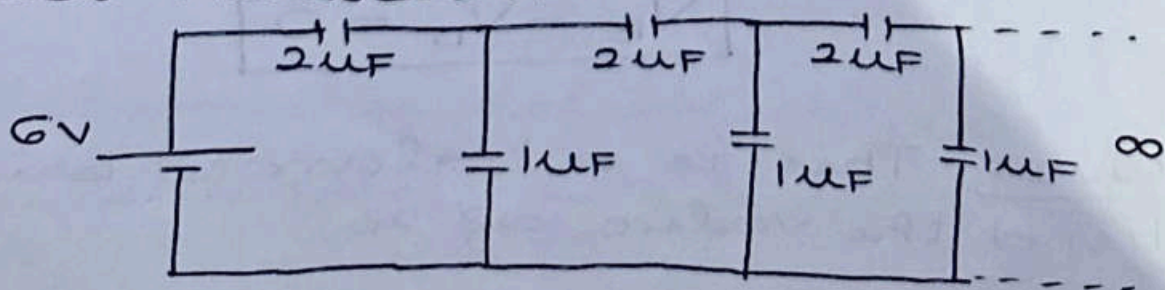


$$C_E = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \dots$$

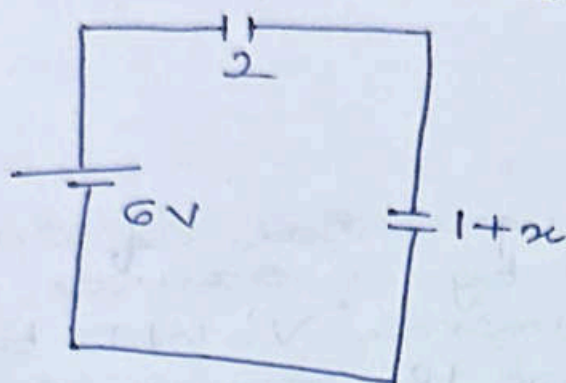
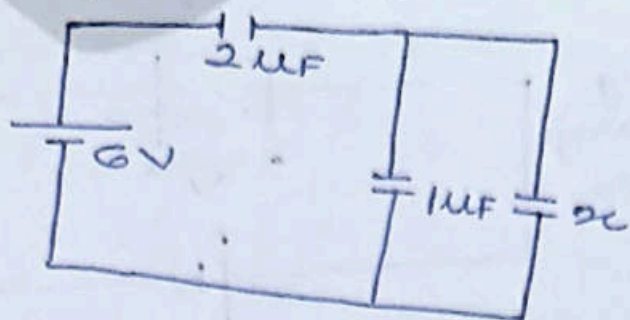
$$C_E = \frac{C}{1 - \frac{1}{2}} = 2C$$

Sum of ∞ G.P.
 $S_n = \frac{a}{1-r}$

Ques Find the capacitance of ∞ ladder network.



Let capacity of ∞ network be ' x '



If any value of capacitor 1 or 2 μF connected in series or in parallel to ∞ capacity, it remain same.

Capacity of ∞ capacitor is 1 μF as capacity can't be negative.

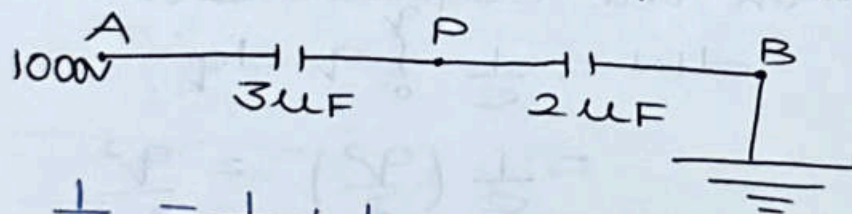
$$\frac{1}{x} = \frac{1}{2} + \frac{1}{1+x}$$

$$\frac{1}{x} = \frac{1+x+2}{2(1+x)}$$

$$x^2 + 2x - x - 2 = 0$$

$$x = 1 \text{ or } -2$$

Ques Find the potential of inner plate of each capacitor.



$$\frac{1}{C} = \frac{1}{3} + \frac{1}{2}$$

$$C = 6/5$$

$$q = CV = \frac{6}{5} \times 1000 = 1200 \mu\text{C}$$

$$\Rightarrow q = C(V_A - V_P) \quad \& \quad q = C(V_P - V_B)$$

$$1200 = 3(100 - V_P) \quad \& \quad 1200 = 2(V_P - 0)$$

$$V_P = 600\text{V}$$

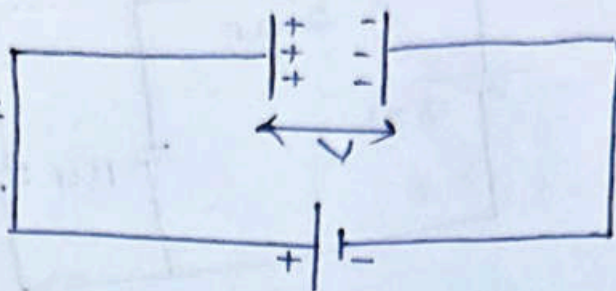
$$\& \quad V_P = 600\text{V}$$

Energy Stored in Capacitor

Work done by source to charge the capacitor:

$$V = \frac{W}{q}$$

$$W = Vq$$



Consider a capacitor of capacity C , charge by a source at potential difference ' V '. W.D. by source to charge the capacitor is stored in the form of electric potential energy.

Small amount of W.D. by source to charge the capacitor

$$\begin{aligned} dW &= V dq \\ &= \frac{q}{C} dq \end{aligned}$$

Total W.D. by the source to store charge q in the capacitor.

$$\begin{aligned} dW &= \frac{1}{C} \int_0^q q dq \\ &= \frac{1}{C} \left(\frac{q^2}{2} \right) = \frac{q^2}{2C} \end{aligned}$$

And this work will be energy stored in the capacitor

$$U = \frac{q^2}{2C}$$

$$[q = CV]$$

$$U = \frac{1}{2} CV^2$$

In capacitor only 50% energy is stored & 50% energy is lost in connecting wire in form of heat.

$$W.D. = \frac{1}{2} CV^2 + H$$

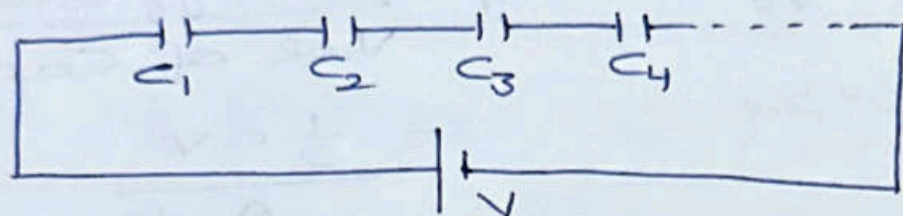
$$\text{Heat Loss } H = \frac{1}{2} CV^2$$

$$W = \frac{1}{2} CV^2 + \frac{1}{2} CV^2$$

$$W = CV^2$$

$$W = qV$$

Energy Stored in Series Combination of Capacitor



$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$$

$$\left[E = \frac{q_s^2}{2C_S} \right]$$

$$E = \frac{q_s^2}{2C_S} = \frac{q_s^2}{2} \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots \right]$$

$$E = \frac{q_s^2}{2C_1} + \frac{q_s^2}{2C_2} + \frac{q_s^2}{2C_3} + \dots$$

$$E = E_1 + E_2 + E_3 + \dots$$

Energy Stored in Parallel Combination of Capacitor

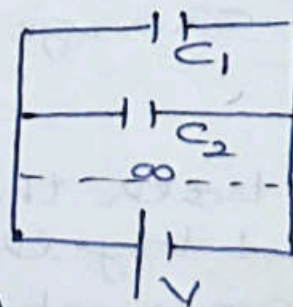
$$C_P = C_1 + C_2 + \dots$$

$$E = \frac{1}{2} C_P V^2$$

$$E = \frac{1}{2} V^2 (C_1 + C_2 + \dots)$$

$$E = \frac{V^2}{2} C_1 + \frac{V^2}{2} C_2 + \dots$$

$$E = E_1 + E_2 + \dots$$



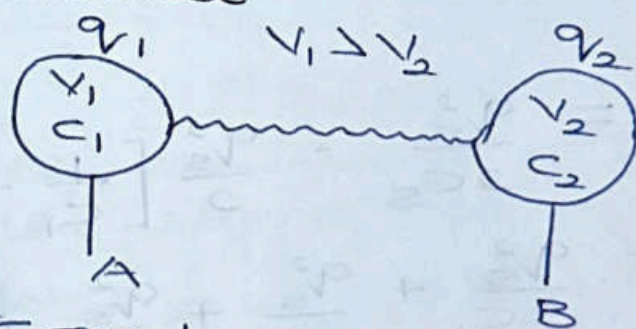
In series and parallel combination of capacitor, energy stored remain same for same value of capacitance and source.

Energy Density of Parallel Plate Capacitor

Energy stored per unit volume of a parallel plate capacitor is called Energy Density.

$$\begin{aligned}\text{Energy Density} &= \frac{\text{Energy Stored}}{\text{Vol. of Capacitor}} \\ &= \frac{\frac{1}{2} C V^2}{A \cdot d} \\ &= \frac{1}{2} \epsilon_0 E^2\end{aligned}$$

Common Potential



Consider two conductors A and B holding a charge q_1 and q_2 at potential V_1 & V_2 . C_1 & C_2 are their capacitance.

Total charge before sharing
 $q' = C_1 V_1 + C_2 V_2$

Now, both the conductors are connected together by a conducting wire and now charge drifts from higher potential to lower potential that is from A to B and a stage that is when potential of both the conductors become same called common potential.

Total charge after sharing

$$q' = C_1 V_1 + C_2 V_2$$

$$q' = (C_1 + C_2) V$$

According to conservation of charge

$$q' = q$$

$$(C_1 + C_2) V = C_1 V_1 + C_2 V_2$$

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$

Where,

V is common potential

Loss of Energy During Charge Sharing

Total initial energy before sharing

$$E_i = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$$

Total final energy after sharing

$$E_f = \frac{1}{2} (C_1 + C_2) V^2$$

$$= \frac{1}{2} (C_1 + C_2) \left[\frac{C_1 V_1 + C_2 V_2}{C_1 + C_2} \right]^2$$

$$= \frac{1}{2} \frac{(C_1 V_1 + C_2 V_2)^2}{(C_1 + C_2)}$$

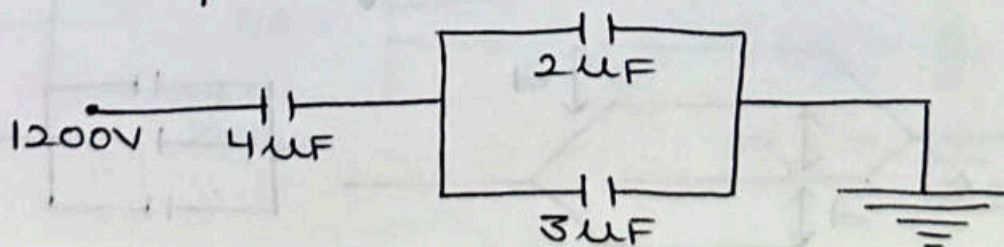
Loss in Energy

$$\Delta E = E_i - E_f$$

$$= \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 - \frac{(C_1 V_1 + C_2 V_2)^2}{2(C_1 + C_2)}$$

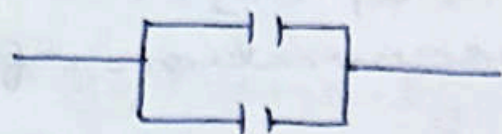
$$= \frac{C_1 C_2 (V_1 - V_2)^2}{2(C_1 + C_2)}$$

Ques Find the charge across each capacitor

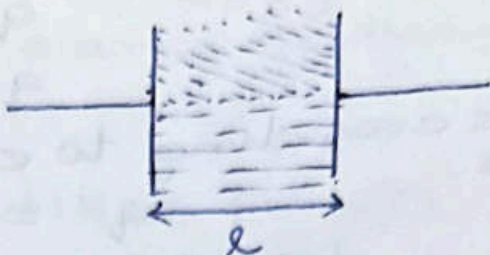


Solving Capacitor in two different medium

(i) Area Division



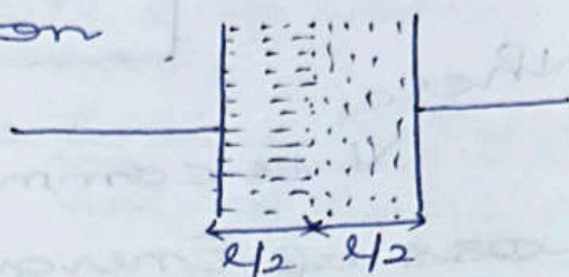
parallel capacitor



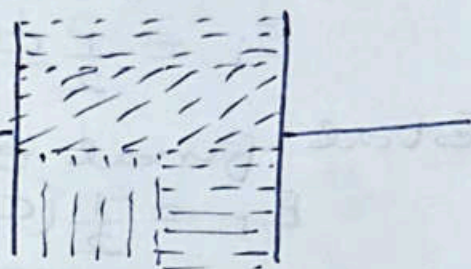
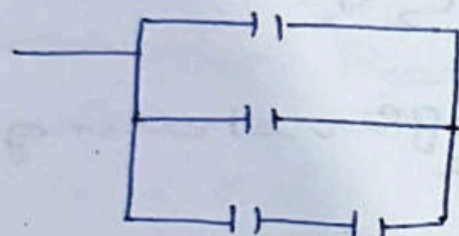
(ii) Distance Division



series capacitor

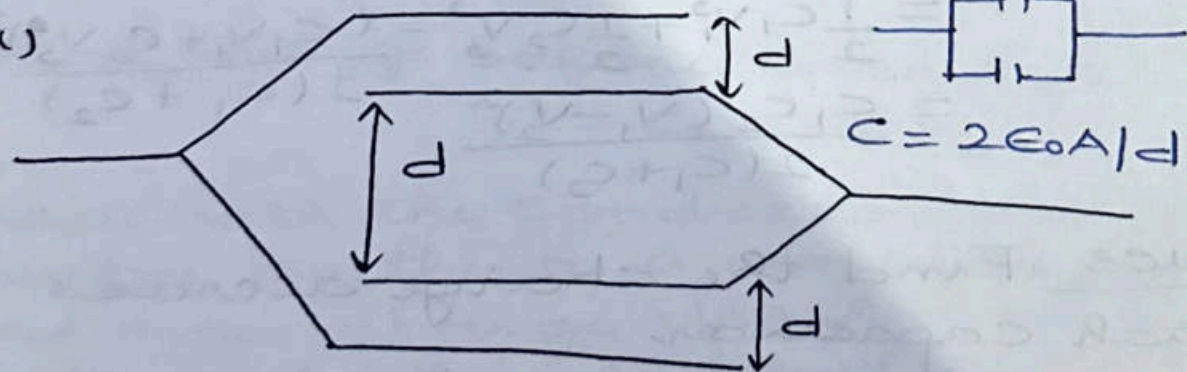


(iii) Both area division as well as distance division

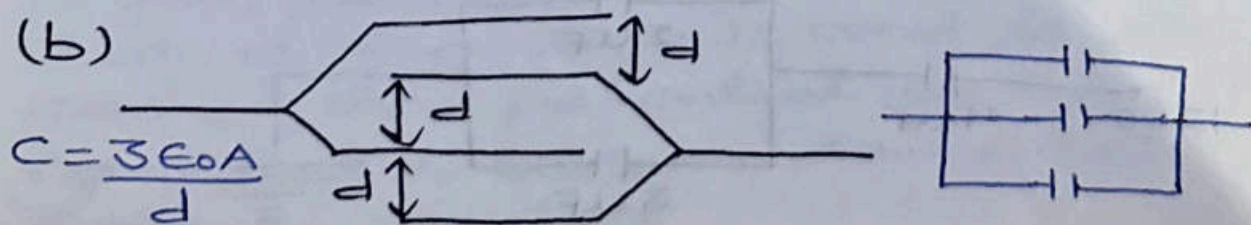


Ques Four Identical plate each has area A as shown in fig. separation b/w consecutive plate capacitor is ' d '. If this arrangement is of parallel plate capacitor. Find equivalent capacitor.

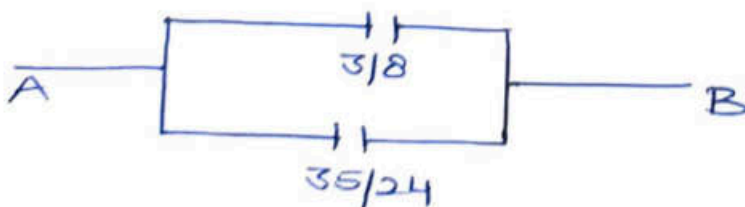
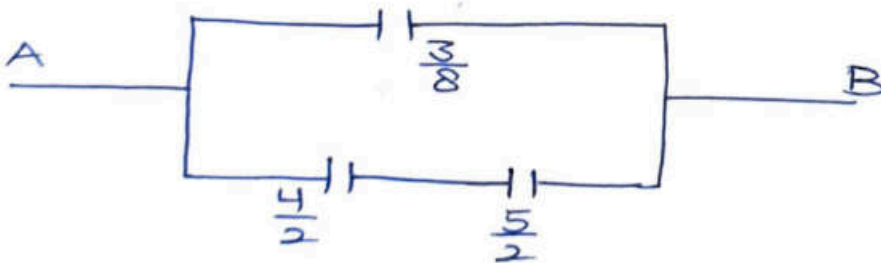
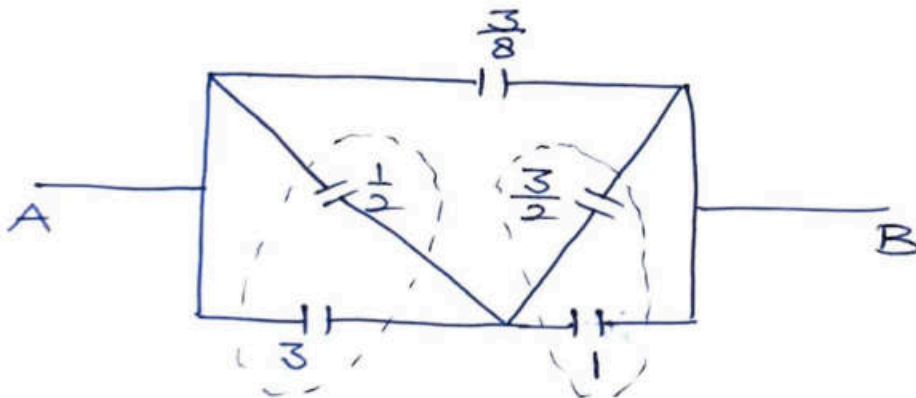
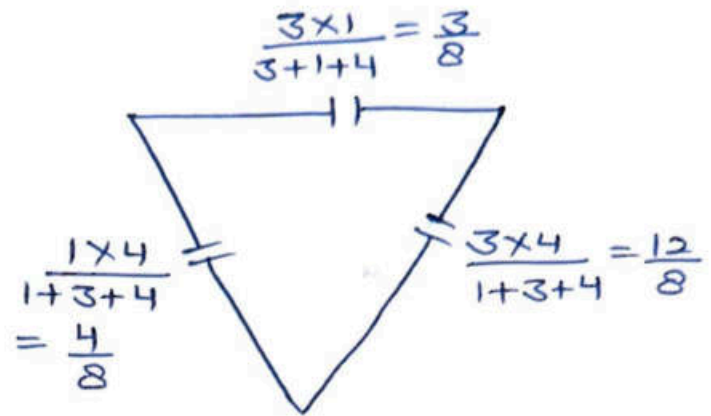
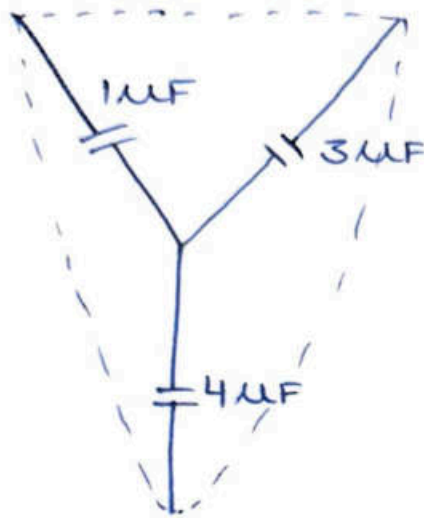
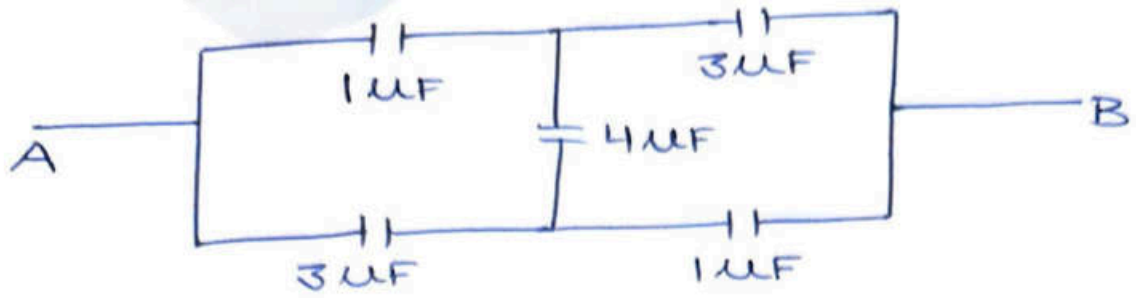
(a)



(b)

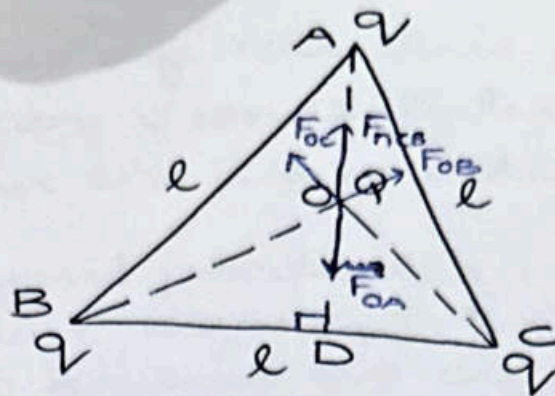


Star Delta Method



$$C_{eq} = \frac{3}{8} + \frac{35}{24} = \frac{11}{6}$$

Ques



$$F_O = ?$$

Ans

$$AD = \sqrt{l^2 - l^2/4} = \sqrt{3}l/2$$

$$AO = \sqrt{3}l/4 \quad AO = \frac{2}{3} \times AD \text{ (because O is centroid)}$$

$$AO = OB = OC \text{ (By symmetry)}$$

$$F_{OA} = \frac{3KQq}{(\sqrt{3}l/4)^2} = F_{OB} = F_{OC}$$

Resultant Force of F_{OB} & F_{OC}

$$F_{ncB} = \sqrt{F_{OB}^2 + F_{OC}^2 + 2F_{OB}F_{OC}\cos\theta}$$

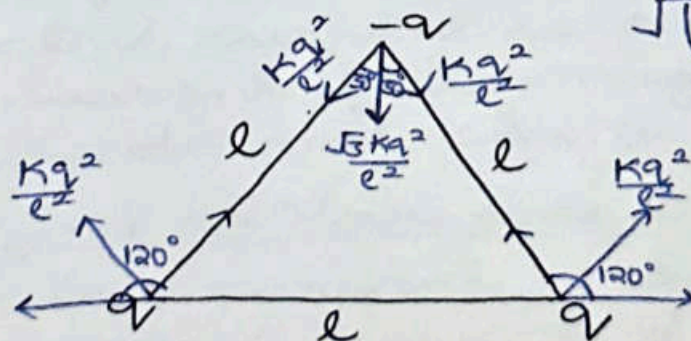
$$\theta = 120^\circ$$

$$F_{ncB} = \sqrt{2F_{OB}^2 - 2 \times \frac{F_{OB}}{2} \times F_{OB}} \\ = F_{OB}$$

Net Force on O

$$F = F_{ncB} - F_{OA} = 0$$

Ques



$$\sqrt{\left(\frac{Kq^2}{l^2}\right)^2 + \left(\frac{Kq^2}{l^2}\right)^2 + \left(\frac{Kq^2}{l^2}\right)^2}$$

$$\sqrt{\left(\frac{Kq^2}{l^2}\right)^2 + \left(\frac{Kq^2}{l^2}\right)^2} = \frac{Kq^2}{l^2}$$

Find force on each charge?

Ans

Current Electricity

The flow of electric charges in a conductor constitutes an electric current in the conductor.

The charged particles which constitute an electric current in solids, liquid, gases are known as current carriers.

Electric current is rate of flow of charges through cross section of a conductor.

$$I = \frac{Q}{t} = \frac{ne}{t}$$

Where, n is no. of e^- through cross section of conductor in time t .

Note-

Electric current is a scalar quantity. Although electric current has magnitude as well as direction yet it is not a vector quantity. It does not require vectors laws and the law of addition of simple algebra are used to find net electric current.

Types of Current

(i) Steady Direct Current

An electric current is said to be steady direct current if its magnitude and direction do not change with time.

(ii) Varying or Variable Direct Current

An electric current is said to be varying direct current if its magnitude change with time and polarity remain same.

(iii) Alternating Current

An electric current is said to be alternating current if its magnitude changes with time and polarity reverses periodically i.e. after regular interval of time.

* The current which does not reverse its polarity is Direct Current (D.C.) eg. Steady current and Varying current of same polarity, whereas the current which reverses its polarity periodically is called Alternating Current.

Drift Velocity

In absence of electric field, average thermal velocity of e^- is zero. When electric field is applied across the conductor all the e^- drift against the electric field.

The velocity with which e^- of a conductor drift toward lower to higher potential or against the electric field called drift velocity or drift speed.

Drift velocity of a electron in a conductor 10^{-3} m/s to 10^{-5} m/s is very-very small but effect is very fast because the electric field produce across the conductor with speed of light i.e. $3 \times 10^8 \text{ m/s}$.

Expression for Drift Velocity

Consider a conductor of length l and area of cross section A , in absence of electric field $u_1, u_2, u_3, \dots, u_n$ are the random velocity of e^- .

$$\frac{\vec{u}_1 + \vec{u}_2 + \vec{u}_3 + \dots + \vec{u}_n}{n} = 0$$

In the presence of electric field all the e^- gain some velocity against the electric field.

$$\begin{aligned}\vec{F} &= m\vec{a} \\ -e\vec{E} &= m\vec{a} \\ \vec{a} &= -\frac{e\vec{E}}{m}\end{aligned}$$

-ve sign show that e^- are against the electric field.

$$V_1 = u_1 + aT_1, \quad V_2 = u_2 + aT_2, \quad \dots$$

$$\text{Drift Velocity } V_d = \frac{V_1 + V_2 + \dots + V_n}{n}$$

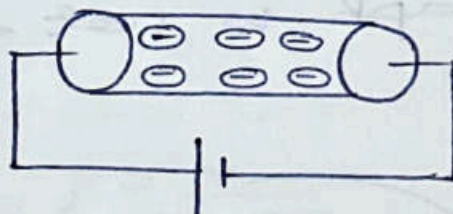
$$V_d = \frac{(u_1 + u_2 + \dots + u_n)}{n} + a \frac{(T_1 + T_2 + \dots + T_n)}{n}$$

$$V_d = \frac{aT}{n}$$

$$\boxed{\vec{V}_d = -\frac{e\vec{E}T}{m}}$$

$$\left[\frac{T_1 + T_2 + \dots + T_n}{n} = T \right]$$

Relation between Current and Drift Velocity



Consider a conductor of length ' l ' and area of cross section ' EA ' and volume of conductor ' EV '.

$$\text{Volume of Conductor} = Al$$

$$n = \text{no. of } e^- \text{ per unit volume}$$

$$\text{Total } e^- = nAl$$

$$\text{Total charge per } e^- \Rightarrow q = neAl$$

$$\text{Current } I = \frac{q}{t} = \frac{neAl}{t} = neAV_d$$

$$\boxed{I = neAV_d}$$

Ohm's Law

Current flowing in a conductor is directly proportional to potential difference develop across it provided some physical condition like temp. remain constant and there is no mechanical strength in the conductor.

$$I \propto V$$

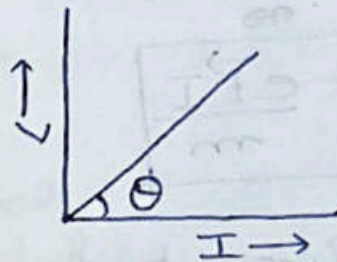
$$V \propto I$$

$$V = IR$$

R = electrical Resistance

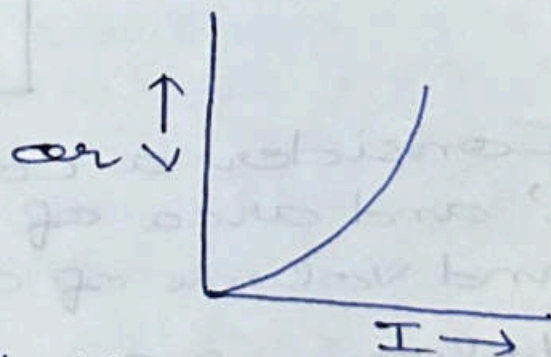
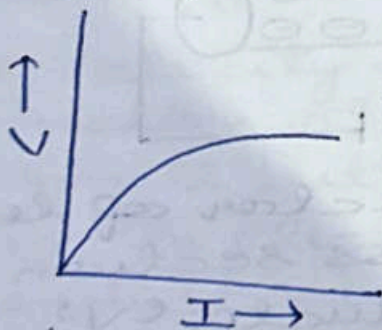
Ohmic Devices

V-I graph is constant for Ohmic Devices. And these are also called Static Resistance.



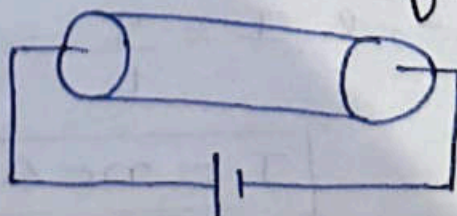
Non-Ohmic Devices

V-I graph is not straight line



Slope of non-ohmic V-I curve give Variable resistance. It is also called Dynamic Resistance.

Derivation of Ohm's Law from Drift Velocity



Consider a conductor of length ' l ' and area of cross section A . When this conductor is connected through source of emf. A potential difference develop accross conductor and all the e^- of the conductor move from $-V_c$ to $+V_c$ potential with velocity V_d .

$$I = neAV_d$$

Put value of V_d

$$I = neA \left(\frac{eE\tau}{m} \right)$$

$$I = \frac{ne^2 A E \tau}{m}$$

$$\left(E = \frac{V}{l} \right)$$

$$I = \frac{ne^2 A V \tau}{m l}$$

$$V = \left(\frac{m l}{ne^2 A \tau} \right) I$$

$$V = R I$$

$$R = \frac{m l}{ne^2 A \tau}$$

$$R \propto l, R \propto \frac{1}{A}, \rho = \frac{m}{ne^2 \tau}$$

ρ is constant for a material and it does not depend upon dimension of conductor i.e. length or area of cross-section.

ρ is property of the material, it is called resistivity or specific resistance.

Note

(I.) If a wire of length l , stretch its length becomes n -times then the new resistance of wire will be

n^2 times.

$$\boxed{R' = n^2 R}$$

PROOF

$$R_1 = \frac{\rho l_1}{A_1}, \quad R_2 = \frac{\rho l_2}{A_2}$$

$$l_2 = n l_1$$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1}$$

$$A_1 l_1 = A_2 l_2$$

$$\frac{A_1}{A_2} = \frac{l_2}{l_1} = n$$

$$\frac{R_1}{R_2} = \frac{l_1}{n l_1} \times \frac{1}{n}$$

$$\boxed{R_2 = n^2 R_1}$$

(2.) A wire of resistance R is stretched and its radius/diameter/thickness become n times then its new resistance will be

$$\boxed{R' = \frac{R}{n^4}}$$

PROOF

$$R_1 = \frac{\rho l_1}{A_1}, \quad R_2 = \frac{\rho l_2}{A_2}$$

$$\frac{R_1}{R_2} = \frac{l_1}{l_2} \times \frac{A_2}{A_1} = \frac{l_1}{l_2} \times \frac{\tau_2^2}{\tau_1^2}$$

$$\tau_2 = n \tau_1$$

$$l_1 A_1 = l_2 A_2$$

$$\frac{l_1}{l_2} = \frac{A_2}{A_1} = \frac{\tau_2^2}{\tau_1^2} = n^2$$

$$\frac{R_1}{R_2} = n^2 \cdot n^2$$

$$R_2 = \frac{R_1}{n^4}$$

Electrical Conductivity

Reciprocal of resistivity of a conductor is called Electrical Conductivity.

$$\sigma = \frac{1}{\rho}$$

Conductance

Reciprocal of electrical resistance is called Conductance.

$$C = \frac{1}{R}$$

Current Density

Current Flowing per unit area of cross section of a conductor is called current Density.

$$\hat{j} = \frac{I}{A}$$

Mobility of Charge Carrier

Drift Velocity per unit electric field is called Mobility of Charge Carrier.

$$\mu = \frac{V_d}{E}$$

Relation between $E, C, \hat{j}$

$$I = neAV_d$$

$$\frac{I}{A} = neV_d$$

$$\hat{j} = \frac{ne^2 EI}{m}$$

$$\boxed{\hat{j} = \sigma E}$$

$$\left(\sigma = \frac{ne^2 \tau}{m} \right)$$

Effect of temperature on Resistance

$$R = \frac{m l}{n e^2 A \tau}$$

τ = Relaxation time

$$R \propto \tau ; R \propto \frac{1}{\tau}$$

When temperature of Resistance Conductor $\uparrow$, Relaxation time $\downarrow$ and resistance $\uparrow$

$$R_t = R_0 (1 + \alpha \Delta T)$$

$$\frac{R_t - R_0}{R_0 \Delta T} = \alpha$$

Where,

α = Temp. constant of Resistance

R_0 = Resistance at 0°C

R_t = Resistance at unknown temp.

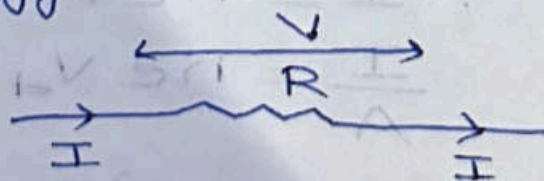
- (i) Metal $\alpha = +ve$ ($T \uparrow$ so $R \uparrow$)
- (ii) Semiconductor $\alpha = -ve$ ($T \uparrow$ so $R \downarrow$)
- (iii) Alloy $\alpha = +ve$ (but α is very small)

Electrical Energy and Power

The work done by source to maintain current in an electric circuit is known as electrical energy.

$$q = I t$$

$$W = V q \\ = V I t$$



This work done is equals to electrical energy consumed in the circuit.

$$E = VIt = I^2 R t = \frac{V^2}{R} t$$

S.I. unit of electrical energy is Joule (J).

Electric Power

It is rate of doing electrical work.

$$P = \frac{E}{t} = VI = I^2 R = \frac{V^2}{R}$$

S.I. unit of Electrical Power is Watt (W)

Cell

A cell is a device which provides necessary potential difference to an electric circuit to maintain a continuous flow of current in it.

EMF (Electromotive Force)

EMF of a cell may be defined as the potential difference between the terminal of the cell when no current is drawn from the cell.

Terminal Potential Difference

Terminal Potential Difference of a cell is defined as the potential difference between its terminals in a closed circuit (i.e. when current is drawn from the cell)

Internal Resistance

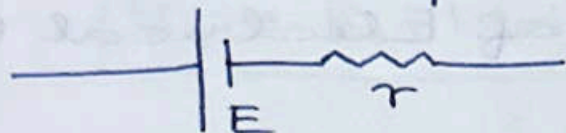
Internal Resistance of a cell is defined as the opposition offered by the electrolyte and electrode of a cell to the flow of current through

it. It is denoted by r and mainly depends on the nature of electrolytes and electrodes of a cell.

Note -

- Direction of conventional current inside the cell is from negative to positive electrode whereas outside the cell it is from positive to negative electrode.

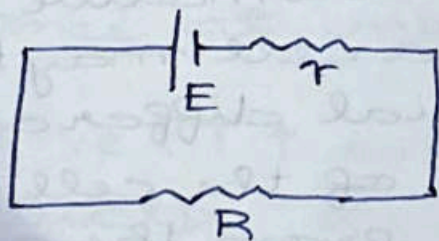
- A cell of EMF ' E ' and internal resistance ' r ' is represented as



- Internal resistance of a cell increases with passage of time and usage of the cell.

- Internal resistance of a cell increases with decrease in temperature.

Relation between terminal potential difference and E.M.F.



Potential difference across terminal of cell is called Terminal Potential Difference.

$$R_{eq} = r + R$$

$$i = \frac{E}{r + R}$$

$$ir + iR = E$$

$$ir + V = E$$

$$\boxed{V = E - ir}$$

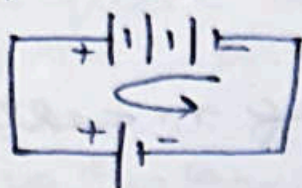
(a) $V < E$

circuit is close

(b) $V = E \Rightarrow i = 0$
circuit is open

(c) $V > E$

This will happen during charging process.



During charging process, direction of cell current is reversed.

$$V = E - (-i)r$$

$$V = E + ir \quad (V > E)$$

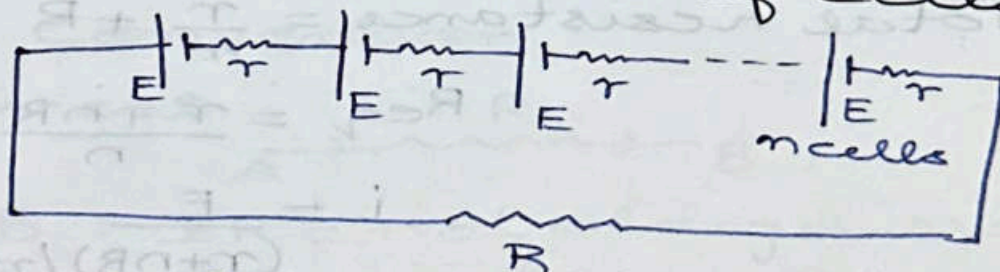
Maximum Power Transfer Theorem
If internal resistance of cell is equal to external resistance of circuit in that situation cell deliver maximum power. So this is called Maximum Power Transfer Theorem.

$$i = \frac{E}{r + R}$$

$$i = \frac{E}{2R} \quad (r = R)$$

$$P_{\max} = I^2 R = \frac{E^2}{4R}$$

Series Combination of cells



Equivalent EMF = nE

Internal Resistance = nr

Total Resistance = $nr + R$

Total current = $\frac{nE}{nr + R}$

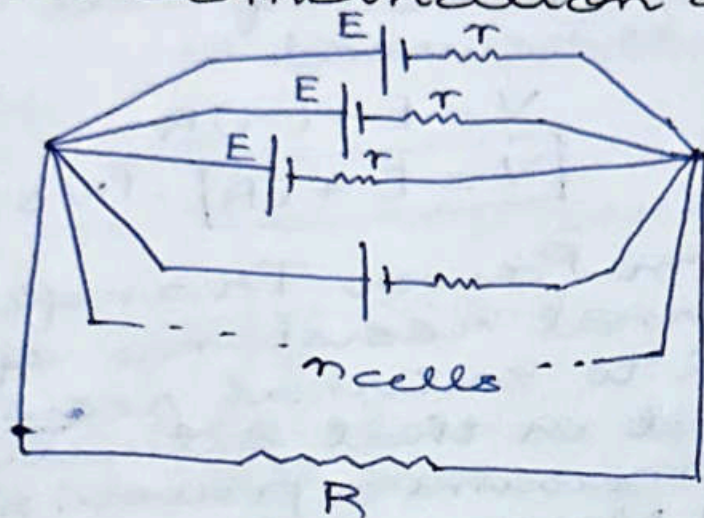
(i) If $n r \gg R$, $R \approx 0$

$$i = \frac{nE}{nr} = \frac{E}{r}$$

(ii) If $n r \ll R$, $nr \approx 0$

$$i = \frac{nE}{R}$$

Parallel Combination of n cells



Cells are said to be in parallel if positive terminal of each cell is connected to a point and negative terminal of each cell is connected to another point.

During this process, arrangement is connected through external resistance R .

$$\text{Total resistance} = \frac{r}{n} + R$$

$$R_{eq} = \frac{r + nR}{n}$$

$$i = \frac{E}{(r + nR)/n}$$

$$i = \frac{nE}{r + nR}$$

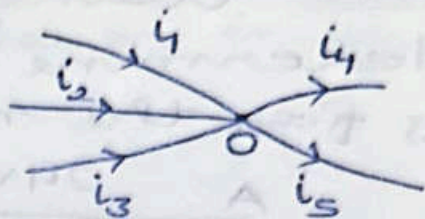
Kirchoff's Law

Junction Law

Algebraic sum of current meeting

at junction is zero.

$$\Sigma I = 0$$



(i) Current coming toward junction is equal to current going away from junction.

(ii) Current coming toward junction is taken as +ve and current going away from junction is taken as -ve.

$$i_1 + i_2 + i_3 - i_4 - i_5 = 0$$

$$\Sigma I = 0$$

Kirchhoff's Voltage Law

In any close loop, algebraic sum of potential difference is equal to zero.

$$\Sigma IR - \Sigma E = 0$$

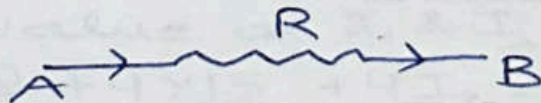
OR

In any close loop, algebraic sum of product of resistance and current is equal to algebraic sum of EMF of that loop.

$$\Sigma IR = \Sigma E$$

Sign Convention

- Current



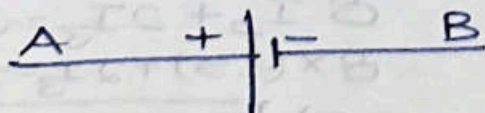
A to B

$IR = +ve$ (along current)

B to A

$IR = -ve$ (opposite to current)

- Cell



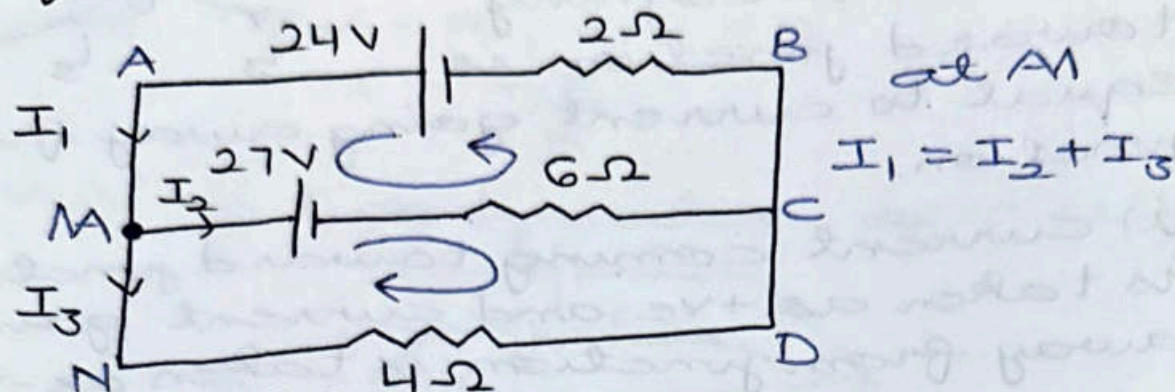
A to B

$E = +ve$

B to A

$E = -ve$

Ques Using Kirchhoff's laws, determine the current I_1 , I_2 and I_3 for the network shown below.



Applying KVL to close loop MCBAM

$$+27 + 6I_2 + 2I_1 - 24 = 0$$

$$2I_1 + 6I_2 = -3 \quad (1)$$

Applying KVL to close loop MCDNM

$$+27 + 6I_2 - 4I_3 = 0$$

$$6I_2 - 4I_3 = -27 \quad (2)$$

$$- (1) + (2)$$

$$-4I_3 - 2I_1 = -24$$

$$I_1 + 2I_3 = 12 \quad (3)$$

put value of I_1 in (1)

$$2I_2 + 2I_3 + 6I_2 = -3$$

$$8I_2 + 2I_3 = -3$$

$$\Rightarrow \begin{aligned} 8I_2 + 2I_3 &= -3 \times 2 \\ 6I_2 - 4I_3 &= -27 \end{aligned}$$

$$16I_2 + 4I_3 = -6$$

$$6I_2 - 4I_3 = -27$$

$$22I_2 = -33$$

$$I_2 = \frac{-33}{22} = -1.5A$$

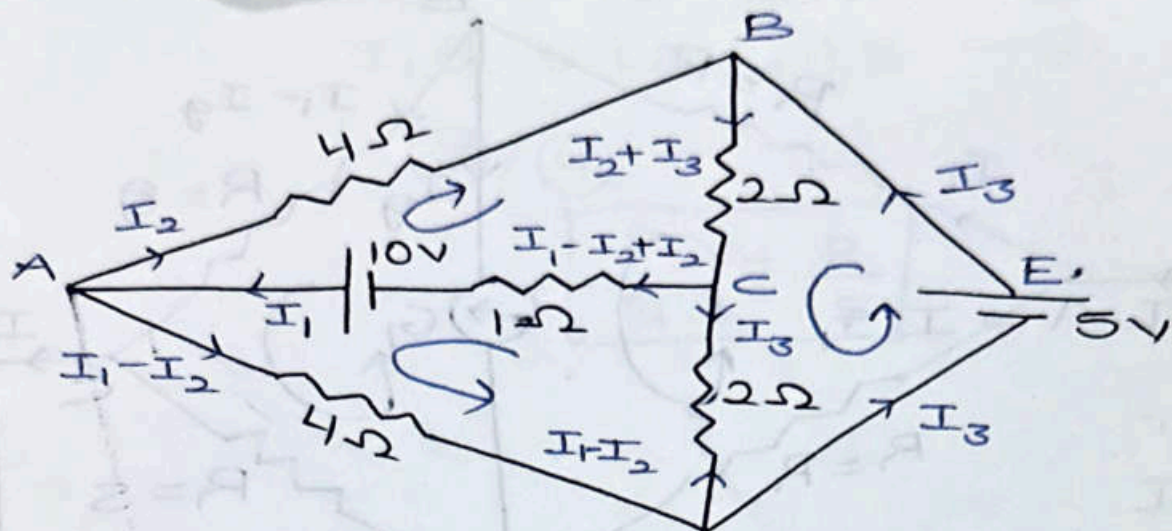
$$8I_2 + 2I_3 = -3$$

$$8 \times (-1.5) + 2I_3 = -3$$

$$\Rightarrow I_3 = 4.5A$$

$$I_1 = I_2 + I_3 = 3A$$

Ques Determine the current in each branch as shown in fig.



Applying KVL to close loop ABCA

$$4I_2 + 2(I_2 + I_3) + 1(I_1 - I_2 + I_2) = 10$$

$$I_1 + 6I_2 + 2I_3 = 10 \quad \text{--- ①}$$

Applying KVL to close loop ADCA

$$4(I_1 - I_2) + 2(I_1 - I_2 - I_3) + 1I_1 - 10 = 0$$

$$7I_1 - 6I_2 - 2I_3 = 10 \quad \text{--- ②}$$

Applying KVL to close loop BCDEB

$$2(I_2 + I_3) + 2(I_3 - I_1 + I_2) = 5$$

$$-2I_1 + 4I_2 + 4I_3 = 5 \quad \text{--- ③}$$

$$\text{①} + \text{②}$$

$$8I_1 = 20$$

$$\boxed{I_1 = 2.5 \text{ A}}$$

$$\text{③} - 2 \times \text{①}$$

$$-8I_2 = -15$$

$$\boxed{I_2 = \frac{15}{8} \text{ A}}$$

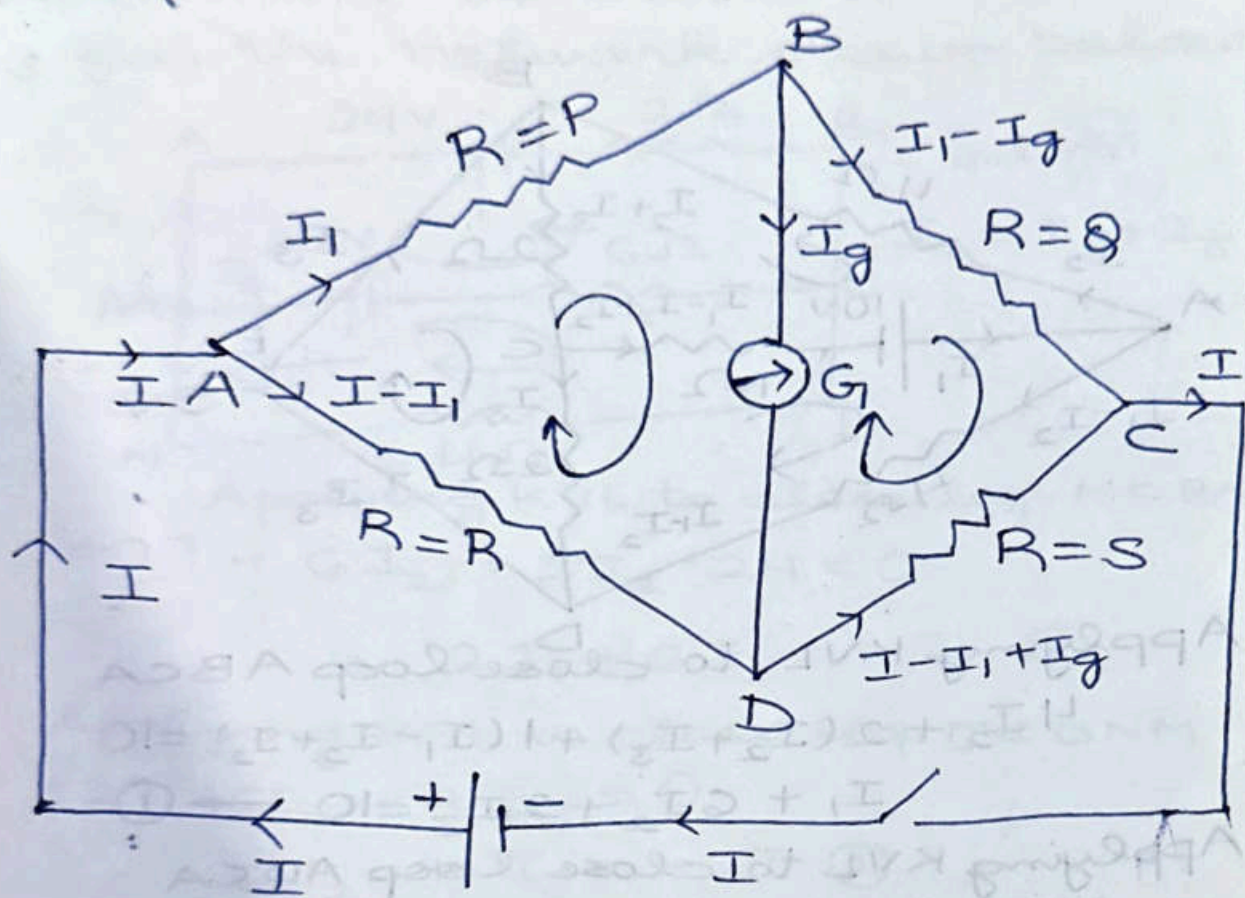
Put value of I_1 & I_2 in ③

$$-2 \times \frac{10}{4} + 4 \times \frac{15}{8} + 4I_3 = 5$$

$$4I_3 = 5 + \frac{5}{2}$$

$$\boxed{I_3 = \frac{5}{8} \text{ A}}$$

Wheat Stone Bridge



Applying KVL in ABDA

$$G I_g + I_1 P - (I - I_1) R = 0$$

$$G I_g + I_1 P = (I - I_1) R \quad \text{--- (1)}$$

Applying KVL in CDBC

$$(I_1 - I_g) Q - (I - I_1 + I_g) S - I_g G = 0$$

$$(I_1 - I_g) Q - I_g G = (I - I_1 + I_g) S \quad \text{--- (2)}$$

Now, value of Resistance, R , at any instant in such a way that on pressing key, Galvanometer show no deflection. In this situation, bridge is in balance condition and no current will flow in galvanometer.

Put $I_g = 0$ in ① & ②

$$I, P = (I - I_1) R \quad \text{--- ③}$$

$$I, \theta = (I - I_1) S \quad \text{--- ④}$$

$$\text{③} / \text{④}$$

$$\boxed{\frac{P}{\theta} = \frac{R}{S}}$$